One-Loop Divergences in Higher-Derivative

高导数中的单圈发散

Gravity

引力

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Abstract

摘要

We give a review of the one-loop divergences in higher-derivative gravity theories. We first make the bilinear expansion in the quantum fluctuation on arbitrary backgrounds, introduce a higher-derivative gauge fixing, and show that higher-derivative gauge fixing must have ghosts in addition to those naively expected. We give general formulae for the one-loop divergences in such theories and give explicit results for theories with quadratic curvature terms. In this calculation, we need the heat kernel coefficients for the four-derivative minimal operators and two-derivative nonminimal vector operators, which are summarized. We also discuss the beta functions in the renormalization group and show that the dimensionless couplings are asymptotically free. The calculation is also extended to the theories with arbitrary functions of R and $R_{\mu\nu}^2$. We show that the result is independent of metric parametrization and gauge on shell.

本文综述高阶导数引力理论中的单圈发散。我们首先在任意背景下对量子涨落做双线性展开，引入高阶导数规范固定，并证明除了原本朴素预期的鬼场外，高阶导数规范固定还必须存在额外的鬼场。我们给出了这类理论中单圈发散的通用公式，并给出了含二次曲率项理论的显式结果。该计算需要四阶导数最小算符和二阶导数非最小矢量算符的热核系数，本文对此进行了总结。我们还讨论了重整化群中的 β 函数，证明无量纲耦合是渐近自由的。本文还将计算推广到含任意 R 和 $R_{\mu\nu}^2$ 函数的理论。我们证明，在壳上该结果与度规参数化和规范选取无关。

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Keywords

关键词

Quantum gravity - Quadratic gravity - Higher-derivative gravity - Perturbation theory - One-loop divergences - Background method - Heat kernel expansion - Effective action - Renormalization group - Asymptotic freedom

量子引力 - 二次引力 - 高阶导数引力 - 微扰理论 - 单圈发散 - 背景场方法 - 热核展开 - 有效作用量 - 重整化群 - 渐近自由

Introduction

引言

The study of quantum effects in gravity started with the seminal work of ’t Hooft and Veltmann [1], in which one-loop divergences were first studied. It was shown that there exist divergences in the quadratic curvature terms, but these counterterms were shown to be transformed away by a field redefinition. However it was shown later the Einstein gravity is nonrenormalizable at two loops [2]. It was further shown that gravity theory containing quadratic curvature terms is a renormalizable theory [3]. Since this theory contains curvature squares and four derivatives, we refer to such theories as quadratic, four-derivative, or higher-derivative gravity. Unfortunately the theory is probably nonunitary in perturbation theory, but see [4] for discussions.

引力中量子效应的研究始于特霍夫特 (’t Hooft) 和韦尔特曼 (Veltmann) 的开创性工作 [1]，该工作首次研究了单圈发散。研究表明二次曲率项中存在发散，但这些抵消项可通过场重定义变换消除。不过后续研究发现，爱因斯坦引力在两圈阶是不可重整的 [2]。进一步研究证明，含二次曲率项的引力理论是可重整理论 [3]。由于该理论包含曲率平方项和四阶导数，我们将这类理论称为二次、四阶导数或高阶导数引力。遗憾的是，该理论在微扰论中可能是非幺正的，相关讨论参见 [4]。

The actual calculation of one-loop divergences in the four-derivative quantum gravity involves some complicated techniques. We use the technique of background field method by separating the quantum fields into backgrounds and fluctuations and integrate over the fluctuations. To really do this at one loop, we have to know the bilinear forms in the fluctuation fields of all relevant terms, such as Weyl curvature square, scalar curvature square (or other combinations, like Ricci curvature square and Riemann curvature square), and Einstein-Hilbert term.

计算四阶导数量子引力中的单圈发散需要用到一些复杂技巧。我们采用背景场方法: 将量子场拆分为背景场和涨落场，随后对涨落积分。要真正完成单圈阶计算，我们需要得到所有相关项中涨落场的双线性形式，包括外尔曲率平方、标量曲率平方 (或其他组合，例如里奇曲率平方、黎曼曲率平方) 以及爱因斯坦-希尔伯特项。

The next step is to gauge fix the theory. Typically this also involves introduction of the Faddeev-Popov (FP) ghosts which were first discovered by Feynman [5], and introduced in more formal way by DeWitt [6], and later elegantly formulated by Faddeev and Popov [7]. When higher-derivative gauge fixing is chosen, this leads to a complication; in addition to those naively expected [8], we have to include additional ghost contribution [denoted as $\mathrm{Tr}\log Y$ in Eq. (1)] first noticed in [9,10] and derived at the one-loop level in [11]. Here instead of examining the one-loop result, we give a more elegant derivation of all the ghosts based on the symmetry principle [12].

下一步是对理论做规范固定。这一步通常还需要引入法捷耶夫-波波夫 (FP) 鬼场: 鬼场最早由费曼发现 [5]，德维特以更形式化的方式引入了它，后来法捷耶夫和波波夫给出了优雅的形式化表述 [7]。如果选择高阶导数规范固定，会带来一个复杂性: 除了那些 naive 预期的贡献 [8]，我们还必须计入额外鬼场贡献 (即式 (1) 中记为 $\mathrm{Tr}\log Y$ 的项)，这一点最早在文献 [9,10] 中被指出，并在文献 [11] 中完成了单圈阶的推导。本文不对单圈结果进行分析，而是基于对称性原理给出所有鬼场的一种更优雅的推导 [12]。

The general formula for the divergences is then schematically given as

发散的一般形式可以概要地写为

$$\Gamma_{\mathrm{div}} = \frac{1}{2}\mathrm{Tr}\log\mathcal{H} - \mathrm{Tr}\log\Delta_{gh} - \frac{1}{2}\mathrm{Tr}\log Y, \tag{1}$$

where the trace is meant to sum over the spectra of the operators, and the first, second, and last terms are the contributions from the graviton, FP ghosts, and additional ghosts, respectively. To calculate this, it is most convenient to use heat kernel technique, which is explained here in some details. The calculation involves fourth-order minimal operators in the tensor sector and second-order nonminimal operators for the ghosts. We give the necessary formulae to evaluate these [13, 14]. Then we sum all the contributions to find the one-loop divergences in the quadratic gravity theory. We mention that there is also the technique called Schwinger-DeWitt method, which is basically the same as the heat kernel technique [15]. There were some mistakes in the early calculations of the logarithmic divergences [8-10], and the correct results were given in [16].

其中迹是对算符的谱求和，第一项、第二项和最后一项分别是引力子、FP 鬼场和额外鬼场的贡献。计算时最方便使用热核技术，本文会对该技术做较为详细的说明。计算过程中，张量部分用到四阶极小算符，鬼场部分用到二阶非极小算符。本文给出计算所需的全部必要公式 [13,14]。随后我们对所有贡献求和，得到二次引力理论中的单圈发散。我们注意到还有一种方法叫施温格-德维特方法，它本质上和热核技术是等价的 [15]。早期对对数发散的计算 [8-10] 存在一些错误，正确结果最早给出在文献 [16] 中。

As a simple application of the technique, we also study general higher-derivative theory including an arbitrary function of Ricci tensor squared and Ricci scalar curvature in a very general parametrization with arbitrary gauge fixing. We will find the general result for this case and show that on shell, it depends on neither the parametrizations nor gauge parameters.

作为该技术的一个简单应用，我们还研究了一般的高阶导数引力理论: 该理论包含里奇张量平方和里奇标量曲率的任意函数，采用非常一般的参数化，搭配任意规范固定。我们将给出该情形的一般结果，并证明在壳上，结果既不依赖参数化也不依赖规范参数。

Bilinear Expansion of Quadratic Terms

二次项的双线性展开

We will consider the Euclidean actions of the general form

我们将研究如下一般形式的欧几里得作用量

$$S=\int d^dx\sqrt{-g}\left[\frac{1}{\kappa^2}(2\Lambda-R)+\alpha R^2+\beta R_{\mu\nu}^2+\gamma R_{\mu\nu\rho\lambda}^2\right], \tag{2}$$

where $\kappa^2=16\pi G$ is the d -dimensional gravitational constant, Λ is the cosmological constant, and α,β,γ are the higher-derivative couplings. Though we consider $d=4$ when we give the results for the divergences, here we give the results for general dimension d . It is sometimes more convenient to use a different basis for the higher-derivative terms, namely, R^2 , the square of the Weyl tensor

其中 $\kappa^2=16\pi G$ 是 d 维引力常数，Λ 是宇宙学常数，α,β,γ 是高阶导数耦合常数。虽然我们给出发散结果时假设维度为 $d=4$ ，但本文会给出一般维度 d 下的结果。对高阶导数项而言，采用另一组基有时会更方便，即 R^2 ，也就是外尔张量的平方

$$C^2=R_{\mu\nu\alpha\beta}^2-\frac{4}{d-2}R_{\mu\nu}^2+\frac{2}{(d-1)(d-2)}R^2, \tag{3}$$

and the Gauss-Bonnet combination

以及高斯-博内组合

$$E=R_{\mu\nu\alpha\beta}^2-4R_{\mu\nu}^2+R^2, \tag{4}$$

which is topological for $d = 4$ and vanishes identically for $d = 3$. Conversely we have

它在 $d = 4$ 维是拓扑项，在 $d = 3$ 维恒等于零。反过来我们有

$$R^2_{\mu\nu\alpha\beta} = \frac{d-2}{d-3}C^2 - \frac{1}{d-3}E + \frac{1}{d-1}R^2, R^2_{\mu\nu} = \frac{d-2}{4(d-3)}\left(C^2 - E\right) + \frac{d}{4(d-1)}R^2. \tag{5}$$

Then the action has the alternative form

因此作用量可以改写为另一种形式

$$S = \int d^d x\sqrt{-g}\left[\frac{1}{\kappa^2}(2\Lambda - R) + \frac{1}{2\lambda}C^2 - \frac{1}{\rho}E + \frac{1}{\xi}R^2\right], \tag{6}$$

where

其中

$$\lambda = \frac{2(d-3)}{(d-2)(\beta+4\gamma)},\ \rho = \frac{4(d-3)}{(d-2)\beta+4\gamma}, \xi = \frac{4(d-1)}{4(d-1)\alpha+d\beta+4\gamma}. \tag{7}$$

or conversely

反过来也可以写成

$$\alpha = -\frac{1}{\rho} + \frac{1}{\xi} + \frac{1}{(d-1)(d-2)\lambda}, \beta = \frac{4}{\rho} - \frac{2}{(d-2)\lambda}, \gamma = -\frac{1}{\rho} + \frac{1}{2\lambda}. \tag{8}$$

Note that in $d = 3, C^2$ and E both vanish identically. The couplings λ, ρ , and ξ have mass dimension $4 - d$. In dimensions higher than three, it is customary to define the dimensionless combinations

注意在 $d = 3, C^2$ 和 E 维，该组合恒等于零。耦合常数 λ, ρ 和 ξ 的质量量纲为 $4 - d$ 。在三维以上维度中，通常会定义如下无量纲组合

$$\omega \equiv -\frac{(d-1)\lambda}{\xi},\ \theta \equiv \frac{\lambda}{\rho}. \tag{9}$$

We will apply the standard background field method, expanding the metric as

我们将采用标准的背景场方法，把度规展开为

$$g_{\mu\upsilon} = \bar{g}_{\mu\upsilon} + h_{\mu\upsilon}. \tag{10}$$

In order to derive the effective action at the one-loop level or to calculate the one-loop beta functions, we need the expansion of the action to second order in $h_{\mu\upsilon}$. For this purpose, it is useful to first make the expansion of curvatures in the fluctuations, which are summarized in Appendix ”Expansion of Curvatures up to Second Order.”

为了推导单圈有效作用量或计算单圈 β 函数，我们需要把作用量展开到 $h_{\mu\upsilon}$ 的二阶。为此，先对曲率按涨落做展开是很有用的，这些展开已总结在附录“曲率到二阶的展开”中。

Using the formulae given in ”Expansion of Curvatures up to Second Order,” we find that the terms proportional to α can be written in the form

利用“曲率到二阶的展开”中给出的公式，我们发现正比于 α 的项可以写为如下形式

$$\alpha h^{\mu\upsilon}\Big[\overline{\nabla}_\mu\overline{\nabla}_\upsilon\overline{\nabla}_\alpha\overline{\nabla}_\beta-\bar{g}_{\mu\upsilon}\Box\overline{\nabla}_\alpha\overline{\nabla}_\beta-\bar{g}_{\alpha\beta}\overline{\nabla}_\mu\overline{\nabla}_\upsilon\Box+\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\Box^2-\bar{R}\bar{g}_{\upsilon\beta}\overline{\nabla}_\alpha\overline{\nabla}_\mu$$

$$-\left(2\bar{R}_{\mu\nu}-\bar{R}\bar{g}_{\mu\nu}\right)\overline{\nabla}_\alpha\overline{\nabla}_\beta+2\bar{R}_{\mu\nu}\bar{g}_{\alpha\beta}\Box+\frac{1}{2}\bar{R}\left(\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta}-\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta}\right)\Box-\bar{R}\bar{R}_{\alpha\beta}\bar{g}_{\mu\nu}$$

$$+2\bar{R}\bar{R}_{\mu\alpha}\bar{g}_{\beta\upsilon}+\bar{R}_{\mu\upsilon}\bar{R}_{\alpha\beta}-\frac{1}{4}J_{\mu\upsilon\alpha\beta}\bar{R}^2\Big]h^{\alpha\beta}\,. \tag{11}$$

Here and in what follows, $\Box\equiv\overline{\nabla}_\mu\overline{\nabla}^\mu$ and a bar indicates that the quantity is evaluated on the background; the indices are raised, lowered, and contracted by the background metric $\bar{g}$, and the covariant derivative $\overline{\nabla}$ is constructed with the background metric. The tensor J is defined by

在下文中，$\Box\equiv\overline{\nabla}_\mu\overline{\nabla}^\mu$ 带横杠表示该物理量是在背景上计算得到的；指标的升、降和缩并都通过背景度规 $\bar{g}$ 完成，协变导数 $\overline{\nabla}$ 也是由背景度规构造的。张量 J 定义为

$$J_{\mu\nu\alpha\beta}=\delta_{\mu\nu,\alpha\beta}-\frac{1}{2}\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta} \tag{12}$$

where

其中

$$\delta_{\mu\upsilon,\alpha\beta}=\frac{1}{2}\left(\bar{g}_{\mu\alpha}\bar{g}_{\upsilon\beta}+\bar{g}_{\mu\beta}\bar{g}_{\upsilon\alpha}\right)\equiv\hat{1} \tag{13}$$

is the identity in the space of symmetric tensors.

是对称张量空间上的恒等算子。

The β terms can be written in the form

β 项可以写为如下形式

$$\beta h^{\mu\upsilon}\Big[\frac{1}{2}\overline{\nabla}_\mu\overline{\nabla}_\upsilon\overline{\nabla}_\alpha\overline{\nabla}_\beta-\frac{1}{2}\bar{g}_{\mu\upsilon}\overline{\nabla}_\alpha\Box\overline{\nabla}_\beta-\frac{1}{2}\bar{g}_{\upsilon\beta}\overline{\nabla}_\mu\Box\overline{\nabla}_\alpha+\frac{1}{4}\left(\bar{g}_{\mu\alpha}\bar{g}_{\upsilon\beta}+\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\right)\Box^2$$

$$+\frac{1}{2}\bar{R}_{\nu\beta}\overline{\nabla}_\alpha\overline{\nabla}_\mu-2\bar{R}^\rho_\mu\bar{g}_{\nu\beta}\overline{\nabla}_\rho\overline{\nabla}_\alpha+\frac{3}{2}\bar{g}_{\alpha\beta}\bar{R}_{\rho\mu}\overline{\nabla}^\rho\overline{\nabla}_\nu+\bar{R}_{\mu\alpha\nu\beta}\Box$$

$$+\frac{1}{4}\left(2\bar{g}_{\mu\alpha}\bar{g}_{\upsilon\beta}-\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\right)\bar{R}^{\rho\lambda}\overline{\nabla}_\rho\overline{\nabla}_\lambda-\frac{3}{2}\bar{R}^\rho_\beta\bar{R}_{\rho\mu\upsilon\alpha}+\bar{R}_{\mu\rho\lambda\upsilon}\bar{R}_\alpha{}^{\rho\lambda}{}_\beta$$

$$+\frac{1}{2}\bar{g}_{\upsilon\beta}\bar{R}_{\mu\rho}\bar{R}^{\rho}_{\alpha}-\bar{g}_{\alpha\beta}\bar{R}_{\mu\rho}\bar{R}^{\rho}_{\upsilon}-\frac{1}{4}J_{\mu\upsilon\alpha\beta}\bar{R}^{2}_{\rho\sigma}]h^{\alpha\beta}, \tag{14}$$

and the terms proportional to γ are

而与 γ 成正比的项为

$$\gamma h^{\mu\upsilon}\Big[\overline{\nabla}_{\mu}\overline{\nabla}_{\nu}\overline{\nabla}_{\alpha}\overline{\nabla}_{\beta}+\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta}\Box^{2}-2\bar{g}_{\upsilon\beta}\overline{\nabla}_{\mu}\Box\overline{\nabla}_{\alpha}-2\bar{g}_{\upsilon\beta}\bar{R}_{\mu\rho\lambda\alpha}\overline{\nabla}^{\rho}\overline{\nabla}^{\lambda}+3\bar{R}_{\mu\alpha\nu\beta}\Box$$

$$-4\bar{R}_{\mu\alpha\rho\nu}\overline{\nabla}^{\rho}\overline{\nabla}_{\beta}-4\bar{g}_{\upsilon\beta}\bar{R}_{\rho\mu}\overline{\nabla}^{\rho}\overline{\nabla}_{\alpha}-2\bar{g}_{\upsilon\beta}\bar{R}_{\mu\alpha}\Box+\bar{g}_{\mu\alpha}\bar{g}_{\upsilon\beta}\bar{R}_{\rho\lambda}\overline{\nabla}^{\rho}\overline{\nabla}^{\lambda}$$

$$-2\bar{g}_{\mu\nu}\bar{R}_{\rho\alpha\lambda\beta}\overline{\nabla}^{\lambda}\overline{\nabla}^{\rho}+4\bar{R}_{\mu\alpha}\overline{\nabla}_{\nu}\overline{\nabla}_{\beta}+2\bar{g}_{\upsilon\beta}\bar{R}_{\mu\lambda\rho\sigma}\bar{R}_{\alpha}{}^{\lambda\rho\sigma}-2\bar{g}_{\upsilon\beta}\bar{R}^{\rho\lambda}\bar{R}_{\mu\rho\alpha\lambda}$$

$$-\bar{R}_{\mu\lambda\rho\beta}\bar{R}_{\upsilon}{}^{\rho\lambda}{}_{\alpha}+3\bar{R}_{\mu\alpha}{}^{\rho\lambda}\bar{R}_{\upsilon\rho\beta\lambda}-3\bar{R}_{\mu\lambda\upsilon\rho}\bar{R}_{\alpha}{}^{\rho\lambda}{}_{\beta}-3\bar{R}^{\rho}_{\alpha}\bar{R}_{\mu\beta\upsilon\rho}$$

$$+2\bar{R}_{\mu\alpha}\bar{R}_{\upsilon\beta}-\bar{g}_{\mu\upsilon}\bar{R}_{\alpha}{}^{\rho\lambda\sigma}\bar{R}_{\beta\rho\lambda\sigma}-\frac{1}{4}J_{\mu\upsilon\alpha\beta}\bar{R}^{2}_{\rho\sigma\lambda\tau}\Big]h^{\alpha\beta}. \tag{15}$$

It can be checked that when arranged in the Gauss-Bonnet combination ($\gamma=\alpha$, $\beta=-4\alpha$) and the background metric is maximally symmetric, one obtains a total derivative. This gives a nontrivial check of the results.

可以验证，当整理为高斯-博内组合 ($\gamma=\alpha$, $\beta=-4\alpha$) 且背景度规是最大对称度规时，最终结果是一个全导数。这是对结果的非平凡检验。

Gauge Fixing and Ghosts

规范固定与鬼场

The first attempt at calculating one-loop divergences was made in [8] with higher-order gauge fixing, but it was pointed out that it did not correctly incorporate the FP ghosts. The problem is that an additional ghost contribution was missing, which was first considered in [9,10]. But it was not clear why we need such additional ghosts and later clarified in [11]; this showed it must be there by carefully examining the one-loop amplitude in the path integral formulation. Here we explain more elegant way of introducing the correct FP ghosts including the additional ghosts for higher-derivative gauge fixing [12], which is valid not only for one loop but also for all loops because it is based on the exact Becchi-Rouet-Stora-Tyutin (BRST) symmetry of the system.

文献 [8] 首次在高阶规范固定下尝试计算单圈发散，但该研究并未正确纳入 FP 鬼场。问题在于它缺失了一项额外鬼场贡献，这一贡献最早由文献 [9,10] 讨论，但当时并不清楚为何需要这类额外鬼场，后来文献 [11] 通过仔细检验路径积分表述下的单圈振幅，明确了该额外鬼场的存在必要性。本文介绍一种更优美的方法，可以引入适用于高阶导数规范固定的正确 FP 鬼场 (包括额外鬼场)[12]，该方法基于体系精确的 Becchi-Rouet-Stora-Tyutin(BRST) 对称性，不仅对单圈成立，对任意圈阶均成立。

The BRST transformation for the fields is found to be

我们得到场的 BRST 变换为

$$\delta_B g_{\mu\nu} = -\delta\lambda\left[g_{\rho\nu}\nabla_\mu c^\rho + g_{\rho\mu}\nabla_\nu c^\rho\right] \equiv -\delta\lambda\mathcal{D}_{\mu\nu,\rho}c^\rho,$$

$$\delta_B c^\mu = -\delta\lambda c^\rho\nabla_\rho c^\mu,\ \delta_B\bar{c}_\mu = i\delta\lambda B_\mu,\ \delta_B B_\mu = 0, \tag{16}$$

which is nilpotent. Here $c^\mu, \bar{c}_\mu$, and B_μ are the FP ghost, anti-ghost, and an auxiliary field, respectively, and $\delta\lambda$ is an anticommuting parameter. The gauge fixing term and the FP ghost terms are concisely written as

该变换是幂零的。此处 $c^\mu, \bar{c}_\mu$ 、B_μ 分别为 FP 鬼场、反鬼场和辅助场，$\delta\lambda$ 是一个反对易参数。规范固定项和 FP 鬼场项可以简洁地写为

$$\mathcal{L}_{GF+FP}/\sqrt{-\bar{g}} = -i\delta_B\left[\bar{c}_\mu Y^{\mu\nu}\left(\chi_\nu - \frac{a}{2}B_\nu\right)\right]/\delta\lambda$$

$$= B_\mu Y^{\mu\nu}\chi_\nu - i\bar{c}_\mu Y^{\mu\nu}\left(\nabla^\lambda\mathcal{D}_{\lambda\nu,\rho} + b\nabla_\nu\mathcal{D}^\lambda_{\lambda,\rho}\right)c^\rho - \frac{a}{2}B_\mu Y^{\mu\nu}B_\nu$$

$$= \frac{1}{2a}\chi_\mu Y^{\mu\nu}\chi_\nu - \frac{a}{2}\widetilde{B}_\mu Y^{\mu\nu}\widetilde{B}_\nu + i\bar{c}_\mu Y^{\mu\nu}\Delta_{gh,\nu\rho}c^\rho \tag{17}$$

where

其中

$$\chi_\mu \equiv \nabla^\lambda h_{\lambda\mu} + b\nabla_\mu h,\ \widetilde{B}_\mu \equiv B_\mu - \frac{1}{a}\chi_\mu, \tag{18}$$

are the gauge fixing function and a field imposing the gauge condition, respectively, $Y^{\mu\nu}$ is a derivative operator for higher-derivative gauge fixing

分别是规范固定函数和施加规范条件的场，$Y^{\mu\nu}$ 是高阶导数规范固定的导数算符

$$Y_{\mu\nu} \equiv -\bar{g}_{\mu\nu}\Box - c\nabla_\mu\nabla_\nu + d\nabla_\nu\nabla_\mu,\ \Delta_{gh,\mu\nu} \equiv -\bar{g}_{\mu\nu}\Box - (1+2b)\nabla_\mu\nabla_\nu - R_{\mu\nu},$$

(19)

$\Delta_{gh,\mu\nu}$ is the ghost kinetic term, and a, b, c , and d are gauge parameters. For one-loop calculation, we can replace ∇ by $\overline{\nabla}$. We see from (17) that we get the determinant factor $(\text{Det } Y^{\mu\nu})^{1/2}$ after we perform the path integral over $\widetilde{B}_\mu$ and FP ghosts $\bar{c}_\mu$ and c^ρ . (The factor $Y^{\mu\nu}$ in the first term combines into the graviton contribution.) In the standard way of Faddeev and Popov, the factor of Y can be easily missed [8], but in this formulation we see why there must be this factor. Note also that the field B_μ is an auxiliary field for low-derivative gauge fixing, but here it becomes dynamical due to the higher-derivative gauge fixing with the factor $Y^{\mu\nu}$.

$\Delta_{gh,\mu\nu}$ 是鬼场动能项，a, b, c 和 d 是规范参数。对于单圈计算，我们可以将 ∇ 替换为 $\overline{\nabla}$ 。由式 (17) 可知，对 $\widetilde{B}_\mu$ 和 FP 鬼场 $\bar{c}_\mu$ 、c^ρ 做路径积分后，我们得到行列式因子 $(\mathrm{Det}\, Y^{\mu\nu})^{1/2}$ 。(第一项中的因子 $Y^{\mu\nu}$ 归入引力子贡献。) 在法捷耶夫-波波夫的标准方法中，Y 因子很容易被遗漏 [8]，但在本文的表述中我们可以明确看到该因子必须存在。还要注意，场 B_μ 在低导数规范固定中是辅助场，但在这里由于带因子 $Y^{\mu\nu}$ 的高阶导数规范固定，它变成动力学场了。

We choose the gauge parameters such that the nonminimal four derivative terms $\overline{\nabla}_\mu\overline{\nabla}_\nu\overline{\nabla}_\alpha\overline{\nabla}_\beta, \bar{g}_{\mu\nu}\Box\overline{\nabla}_\alpha\overline{\nabla}_\beta$, and $\bar{g}_{\nu\beta}\overline{\nabla}_\mu\Box\overline{\nabla}_\alpha$ cancel. This leads to the choice [18,19]

我们选取规范参数，使非最小四导数项 $\overline{\nabla}_\mu\overline{\nabla}_\nu\overline{\nabla}_\alpha\overline{\nabla}_\beta, \bar{g}_{\mu\nu}\Box\overline{\nabla}_\alpha\overline{\nabla}_\beta$ 和 $\bar{g}_{\nu\beta}\overline{\nabla}_\mu\Box\overline{\nabla}_\alpha$ 抵消，得到的选择为 [18,19]

$$a = \frac{1}{\beta + 4\gamma},\ b = \frac{4\alpha + \beta}{4(\gamma - \alpha)},\ c - d = \frac{2(\gamma - \alpha)}{\beta + 4\gamma} - 1. \tag{20}$$

In order to simplify the gauge fixing term, we will further choose $d = 1$. Then, the quadratic terms in the action can be written in the form $h^{\mu\upsilon}\mathcal{K}_{\mu\upsilon,\alpha\beta}h^{\alpha\beta}$, where

为简化规范固定项，我们进一步选取 $d = 1$ 。此时作用量中的二次项可以写为 $h^{\mu\upsilon}\mathcal{K}_{\mu\upsilon,\alpha\beta}h^{\alpha\beta}$ 的形式，其中

$$\mathcal{K} = K\Box^2 + D_{\rho\lambda}\overline{\nabla}^\rho\overline{\nabla}^\lambda + W. \tag{21}$$

The explicit forms of the coefficients are

系数的具体形式为

$$(K)_{\mu\upsilon,\alpha\beta} = \frac{\beta + 4\gamma}{4}\left(\bar{g}_{\mu\alpha}\bar{g}_{\upsilon\beta} + \frac{4\alpha + \beta}{4(\gamma - \alpha)}\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\right), \tag{22}$$

$$\begin{aligned}
&(D_{\rho\lambda})_{\mu\nu,\alpha\beta} \\
&= -2\gamma\bar{g}_{\upsilon\beta}\bar{R}_{\alpha\rho\lambda\mu} + 4\gamma\bar{g}_{\rho\upsilon}\bar{R}_{\lambda\alpha\mu\beta} + (\beta + 3\gamma)\bar{g}_{\rho\lambda}\bar{R}_{\mu\alpha\upsilon\beta} - (2\beta + 4\gamma)\bar{g}_{\alpha\rho}\bar{g}_{\upsilon\beta}\bar{R}_{\mu\lambda} \\
&-2\gamma\bar{g}_{\upsilon\beta}\bar{R}_{\mu\alpha}\bar{g}_{\rho\lambda} + \beta\bar{g}_{\mu\upsilon}\bar{g}_{\beta\rho}\bar{R}_{\alpha\lambda} - 2\alpha\bar{g}_{\alpha\rho}\bar{g}_{\beta\lambda}\bar{R}_{\mu\upsilon} + 2\alpha\bar{g}_{\mu\upsilon}\bar{g}_{\rho\lambda}\bar{R}_{\alpha\beta} + 2\gamma\bar{g}_{\mu\upsilon}\bar{R}_{\alpha\rho\lambda\beta} \\
&+\left(\frac{\alpha}{2}\bar{R} - \frac{1}{4\kappa^2}\right)\left(\bar{g}_{\mu\alpha}\bar{g}_{\upsilon\beta}\bar{g}_{\rho\lambda} - \bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\bar{g}_{\rho\lambda} - 2\bar{g}_{\upsilon\beta}\bar{g}_{\mu\rho}\bar{g}_{\alpha\lambda} + 2\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\rho}\bar{g}_{\beta\lambda}\right) \\
&+2\gamma\bar{g}_{\upsilon\rho}\bar{g}_{\beta\lambda}\bar{R}_{\mu\alpha} + \left(\frac{\beta}{2} + \gamma\right)\bar{g}_{\mu\alpha}\bar{g}_{\upsilon\beta}\bar{R}_{\rho\lambda} - \frac{\beta}{4}\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\bar{R}_{\rho\lambda},
\end{aligned} \tag{23}$$

$$\begin{aligned}
&(W)_{\mu\nu,\alpha\beta} \\
&= \frac{3}{2}\gamma\bar{g}_{\upsilon\beta}\bar{R}_\mu{}^{\rho\lambda\sigma}\bar{R}_{\alpha\rho\lambda\sigma} + 4\gamma\bar{R}_{\rho\alpha\mu\lambda}\bar{R}_{\upsilon\beta}{}^{\rho\lambda} - \gamma\bar{R}_{\rho\alpha\mu\lambda}\bar{R}^\rho{}_{\upsilon\beta}{}^\lambda + (\beta + 5\gamma)\bar{R}_{\rho\mu\lambda\upsilon}\bar{R}^\rho{}_\alpha{}^\lambda{}_\beta
\end{aligned}$$

$$+6\gamma\bar{R}^{\rho}_{\mu}\bar{R}_{\rho\alpha\beta\upsilon}+\left(\frac{\beta}{2}+\gamma\right)\bar{R}_{\mu\alpha}\bar{R}_{\upsilon\beta}+\left(\alpha\bar{R}-\frac{1}{2\kappa^2}\right)\left(\frac{1}{2}\bar{R}_{\mu\alpha\upsilon\beta}+\frac{3}{2}\bar{g}_{\beta\upsilon}\bar{R}_{\mu\alpha}-\bar{g}_{\mu\upsilon}\bar{R}_{\alpha\beta}\right)$$

$$+\alpha\bar{R}_{\mu\nu}\bar{R}_{\alpha\beta}+\frac{1}{8}\left(\alpha\bar{R}^2+\beta\bar{R}^2_{\rho\lambda}+\gamma\bar{R}^2_{\rho\lambda\sigma\tau}-\frac{1}{\kappa^2}\left(\bar{R}-2\Lambda\right)\right)\left(\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta}-2\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta}\right)$$

$$+\left(\frac{5}{2}\beta+4\gamma\right)\bar{g}_{\nu\beta}\bar{R}_{\mu\rho}\bar{R}^{\rho}_{\alpha}-\gamma\bar{g}_{\mu\nu}\bar{R}_{\alpha}{}^{\rho\lambda\sigma}\bar{R}_{\beta\rho\lambda\sigma}-\beta\bar{g}_{\alpha\beta}\bar{R}_{\mu\rho}\bar{R}^{\rho}_{\nu}-(\beta+4\gamma)\,\bar{g}_{\nu\beta}\bar{R}^{\rho\lambda}\bar{R}_{\mu\rho\alpha\lambda},$$

(24)

where we have dropped terms with two derivatives acting on a background curvature and performed the symmetrizations $\mu\leftrightarrow\upsilon,\alpha\leftrightarrow\beta$, and $(\mu,\upsilon)\leftrightarrow(\alpha,\beta)$.

其中我们舍去了作用在背景曲率上的二阶导数项，并完成了对称化 $\mu\leftrightarrow\upsilon,\alpha\leftrightarrow\beta$ 和 $(\mu,\upsilon)\leftrightarrow(\alpha,\beta)$ 。

In order to use the heat kernel formula, we have to put this operator into the form

为了使用热核公式，我们需要将该算符整理为以下形式

$$\mathcal{H}=K^{-1}\mathcal{K}=\Box^2+V_{\rho\lambda}\overline{\nabla}^{\rho}\overline{\nabla}^{\lambda}+U, \tag{25}$$

where

其中

$$\left(K^{-1}\right)_{\mu\upsilon}{}^{\alpha\beta}=\frac{4}{\beta+4\gamma}\left(\delta_{\mu\upsilon}{}^{\alpha\beta}-\Omega\bar{g}_{\mu\upsilon}\bar{g}^{\alpha\beta}\right), \tag{26}$$

with

满足

$$\Omega=\frac{4\alpha+\beta}{\sum},\ \sum\equiv 4\left(\gamma-\alpha\right)+d\left(4\alpha+\beta\right). \tag{27}$$

The form of the coefficients $V_{\rho\lambda}$ and U is complicated. First V is given by

系数 $V_{\rho\lambda}$ 和 U 的形式较为复杂。首先 V 由下式给出

$$V^{\rho\lambda}=\frac{4}{\beta+4\gamma}\sum_{i=1}^{20}b_i\mathbf{k}_i \tag{28}$$

where

其中

$$\mathbf{k}_1=\bar{g}_{\upsilon\beta}\bar{g}^{\rho\lambda}\bar{R}_{\mu\alpha},\ \mathbf{k}_2=\delta_{\mu\upsilon,\alpha\beta}\bar{g}^{\rho\lambda},\ \mathbf{k}_3=\bar{g}^{\rho\lambda}\bar{R}_{\mu\alpha\upsilon\beta},\ \mathbf{k}_4=\delta_{\upsilon\beta}{}^{\rho\lambda}\bar{R}_{\mu\alpha},$$

$$\mathbf{k}_5=\delta_{\nu\beta}{}^{\rho\lambda}\bar{g}_{\mu\alpha},\ \mathbf{k}_6=\delta_{\mu\nu,\alpha\beta}\bar{R}^{\rho\lambda},\ \mathbf{k}_7=\frac{1}{2}\left(\delta_\nu^{(\rho}\bar{R}^{\lambda)}{}_{\alpha\beta\mu}+\delta_\beta^{(\rho}\bar{R}^{\lambda)}{}_{\mu\nu\alpha}\right),$$

$$\mathbf{k}_8=\bar{g}_{\upsilon\beta}\delta^{(\rho}_{(\mu}\bar{R}^{\lambda)}_{\alpha)},\ \mathbf{k}_9=\bar{g}_{\upsilon\beta}\bar{R}_{(\alpha}{}^{\rho\lambda}{}_{\mu)},\ \mathbf{k}_{10}=\frac{1}{2}\left(\delta_{\alpha\beta}{}^{\rho\lambda}\bar{R}_{\mu\upsilon}+\delta_{\mu\upsilon}{}^{\rho\lambda}\bar{R}_{\alpha\beta}\right),$$

$$\mathbf{k}_{11}=\bar{g}_{\mu\upsilon}\bar{R}_\alpha{}^{\rho\lambda}{}_\beta,\ \mathbf{k}_{12}=\bar{g}_{\alpha\beta}\bar{R}_\mu{}^{\rho\lambda}{}_\upsilon,\ \mathbf{k}_{13}=\bar{g}_{\mu\upsilon}\bar{g}^{\rho\lambda}\bar{R}_{\alpha\beta},\ \mathbf{k}_{14}=\bar{g}_{\alpha\beta}\bar{g}^{\rho\lambda}\bar{R}_{\mu\upsilon},$$

$$\mathbf{k}_{15}=\bar{g}_{\mu\nu}\delta^\lambda_\alpha\bar{R}^\rho_\beta,\ \mathbf{k}_{16}=\bar{g}_{\alpha\beta}\delta^\lambda_\mu\bar{R}^\rho_\upsilon,\ \mathbf{k}_{17}=\bar{g}_{\mu\upsilon}\delta_{\alpha\beta}{}^{\rho\lambda},\ \mathbf{k}_{18}=\bar{g}_{\alpha\beta}\delta_{\mu\upsilon}{}^{\rho\lambda},$$

$$\mathbf{k}_{19}=\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\bar{g}^{\rho\lambda},\ \mathbf{k}_{20}=\bar{g}_{\mu\upsilon}\bar{g}_{\alpha\beta}\bar{R}^{\rho\lambda}, \tag{29}$$

and

且

$$b_1=-2\gamma,\ b_2=\frac{\alpha}{2}\bar{R}-\frac{1}{4\kappa^2},\ b_3=\beta+3\gamma,\ b_4=2\gamma,\ b_5=\frac{1}{2\kappa^2}-\alpha\bar{R},$$

$$b_6=\frac{\beta}{2}+\gamma,\ b_7=-4\gamma,\ b_8=-2\beta-4\gamma,\ b_9=-2\gamma,\ b_{10}=-2\alpha,$$

$$b_{11}=4\gamma\Omega_3,\ b_{12}=\gamma,\ b_{13}=-\beta\Omega_3,\ b_{14}=\alpha,\ b_{15}=2\beta\Omega_3,\ b_{16}=\frac{\beta}{2},$$

$$b_{17}=2\alpha\Omega_3\bar{R}-\frac{\Omega_1-2\Omega}{2\kappa^2},\ b_{18}=\frac{\alpha}{2}\bar{R}-\frac{1}{4\kappa^2},\ b_{19}=-b_{17},\ b_{20}=-\beta\Omega_3.$$

(30)
where

其中

$$\Omega_1=\frac{10\alpha+3\beta+2\gamma}{\sum},\ \Omega_3=\frac{3\alpha+\beta+\gamma}{\sum}, \tag{31}$$

with $\sum$ given in (27). Next,

$\sum$ 由式 (27) 给出。接下来,

$$(U)_{\mu\nu,\alpha\beta}$$

$$=\frac{4}{\beta+4\gamma}\left[\frac{3}{2}\gamma\bar{g}_{\upsilon\beta}\bar{R}_\mu{}^{\rho\lambda\sigma}\bar{R}_{\alpha\rho\lambda\sigma}-\gamma\bar{R}^\lambda{}_{\alpha\mu}{}^\rho\bar{R}_{\lambda\upsilon\beta\rho}+4\gamma\bar{R}_{\rho\alpha\mu\lambda}\bar{R}_{\upsilon\beta}{}^{\rho\lambda}\right.$$

$$-3\gamma\left(\bar{R}^\sigma_\mu\bar{R}_{\sigma\alpha\nu\beta}+\bar{R}^\sigma_\alpha\bar{R}_{\sigma\mu\beta\nu}\right)+\left(\frac{\beta}{2}+\gamma\right)\bar{R}_{\mu\alpha}\bar{R}_{\nu\beta}-\frac{\gamma}{2}\bar{g}_{\alpha\beta}\bar{R}_{\mu\rho\lambda\sigma}\bar{R}_\nu{}^{\rho\lambda\sigma}$$

$$+\frac{1}{4}S^2\left(\Omega_1\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta}-\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta}\right)+\left(\frac{\alpha}{2}\bar{R}-\frac{1}{4\kappa^2}\right)\left(\bar{R}_{\mu\alpha\nu\beta}+3\bar{g}_{\nu\beta}\bar{R}_{\mu\alpha}-\bar{g}_{\alpha\beta}\bar{R}_{\mu\nu}\right)$$

$$+\left(\frac{5}{2}\beta+4\gamma\right)\bar{g}_{\nu\beta}\bar{R}_{\mu\sigma}\bar{R}^{\sigma}_{\alpha}+(\beta+5\gamma)\,\bar{R}_{\rho\mu\lambda\nu}\bar{R}^{\rho}{}_{\alpha}{}^{\lambda}{}_{\beta}-\frac{\beta}{2}\bar{g}_{\alpha\beta}\bar{R}_{\mu\sigma}\bar{R}^{\sigma}_{\nu}$$

$$-\gamma\Omega_1\bar{g}_{\mu\nu}\bar{R}_{\alpha\rho\lambda\sigma}\bar{R}_{\beta}{}^{\rho\lambda\sigma}-\beta\Omega_1\bar{g}_{\mu\nu}\bar{R}_{\alpha\sigma}\bar{R}^{\sigma}_{\beta}+\alpha\bar{R}_{\mu\nu}\bar{R}_{\alpha\beta}+\left(\frac{1}{\kappa^2}\Omega_3-\alpha\Omega_1\bar{R}\right)\bar{g}_{\mu\nu}\bar{R}_{\alpha\beta}$$

$$+\frac{1}{4\kappa^2}\left(\bar{R}-4\Lambda\right)\Omega\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta}-(\beta+4\gamma)\,\bar{g}_{\nu\beta}\bar{R}^{\rho\lambda}\bar{R}_{\mu\rho\alpha\lambda}\Bigg], \tag{32}$$

where we have defined

我们在此定义

$$S^2=\alpha\bar{R}^2+\beta\bar{R}^2_{\mu\nu}+\gamma\bar{R}^2_{\mu\nu\rho\lambda}-\frac{1}{\kappa^2}\left(\bar{R}-2\Lambda\right). \tag{33}$$

These results are given in Refs. [18, 19].

这些结果见文献 [18, 19]。

General Formula for One-Loop Divergences

单圈发散的一般公式

Correcting the errors in [8], the results for one-loop divergences were given in [9, 10], but they still contained errors, and this was corrected in [16]. Here we give the correct results using heat kernel expansion.

修正文献 [8] 中的错误后，文献 [9, 10] 给出了单圈发散的结果，但仍存在错误，这些错误在文献 [16] 中得到了修正。本文我们利用热核展开给出正确结果。

The partition function for the one loop is obtained from the quadratic terms of the action as

单圈配分函数可由作用量的二次项得到，形式为

$$Z=\mathrm{Det}\left(\Delta\right)^{-1/2} \tag{34}$$

for a real scalar field. and the effective action is given by

对应实标量场，有效作用量由下式给出

$$\Gamma=-\log Z=\frac{1}{2}\,\mathrm{Tr}\log\Delta. \tag{35}$$

If the fluctuations are anticommuting fields like the FP ghosts, the sign should be opposite. If the fluctuations are complex fields (or independent two hermitian fields), the front factor should be 1 .

如果涨落是 FP 鬼这类反对易场，符号应取反。如果涨落是复场 (或两个独立的厄米场)，前置因子应为 1。

Suppose we know the eigenvalues and eigenfunctions of the operator Δ :

假设我们已知算符 Δ 的本征值和本征函数:

$$\Delta\phi_n = \lambda_n\phi_n \tag{36}$$

Then we can evaluate (35) as

我们可以将 (35) 计算为

$$\frac{1}{2}\operatorname{Tr}\log\Delta = \frac{1}{2}\sum_n \log\lambda_n \tag{37}$$

We define the zeta function for Δ :

我们为 Δ 定义 ζ 函数:

$$\zeta_\Delta(s) = \sum_{n=1}^{\infty}\lambda_n^{-s} = \frac{1}{\Gamma(s)}\int_0^\infty dt t^{s-1}\operatorname{Tr}\left(e^{-t\Delta}\right), \tag{38}$$

and obtain

得到

$$\frac{1}{2}\operatorname{Tr}\log\Delta = -\frac{1}{2}\frac{d}{ds}\zeta_\Delta(s)\Big|_{s=0} = -\frac{1}{2}\frac{d}{ds}\left[\frac{1}{\Gamma(s)}\int_0^\infty dt t^{s-1}\operatorname{Tr}\left(e^{-t\Delta}\right)\right]_{s=0} \qquad (.\ 39)$$

For a differential operator of order p in d dimensions, the heat kernel $e^{-t\Delta}$ has the expansion

对于 d 维空间中阶数为 p 的微分算符，热核 $e^{-t\Delta}$ 满足以下展开式

$$\operatorname{Tr}\left(e^{-t\Delta}\right) = \int\frac{d^d x}{(4\pi)^{d/2}}\sqrt{g}\sum_{n=0}^{\infty} b_{2n}(\Delta)\, t^{(2n-d)/p}. \tag{40}$$

Typically we have the operators of $p=4$ and $p=2$. The formulae are given separately. The divergent part of the effective action for $p=4$ operator is evaluated as

通常我们遇到 $p=4$ 阶和 $p=2$ 维的算符，下文分别给出公式。$p=4$ 算符对应有效作用量的发散部分计算结果为

$$\begin{aligned}\Gamma^{(4)} &= -\frac{1}{2}\int\frac{d^d x}{(4\pi)^{d/2}}\sqrt{g}\int_{1/\Lambda_{\mathrm{UV}}^4}^{1/\mu^4} dt\left[t^{-\frac{d}{4}-1}b_0 + t^{-\frac{d-2}{4}-1}b_2 + \cdots + t^{-1}b_d + \cdots\right]\\ &= -\frac{1}{2}\int\frac{d^d x}{(4\pi)^{d/2}}\sqrt{g}\left[\frac{\Lambda_{\mathrm{UV}}^d}{d/4}b_0 + \frac{\Lambda_{\mathrm{UV}}^{d-2}}{(d-2)/4}b_2 + \cdots + \log\frac{\Lambda_{\mathrm{UV}}^4}{\mu^4}b_d + \text{ finite }\right],\end{aligned}$$

(41)

where the heat kernel coefficients are given in section "Contributions from $p = 4$ Minimal Operator." Here Λ_{UV} is an ultraviolet cutoff which should be distinguished from the cosmological constant Λ .

其中热核系数在“$p=4$ 最小算符的贡献”一节给出。此处 Λ_{UV} 是紫外截断，需与宇宙学常数 Λ 区分开。

The divergent part of the effective action for $p = 2$ operator is given by

$p=2$ 算符对应有效作用量的发散部分由下式给出

$$
\begin{aligned}
\Gamma^{(2)} &= -\frac{1}{2}\int \frac{d^d x}{(4\pi)^{d/2}}\sqrt{g}\int_{1/\Lambda_{\mathrm{UV}}^2}^{1/\mu^2} dt\left[t^{-\frac{d}{2}-1}b_0 + t^{-\frac{d}{2}}b_2 + \cdots + t^{-1}b_d + \cdots\right] \\
&= -\frac{1}{2}\int \frac{d^d x}{(4\pi)^{d/2}}\sqrt{g}\left[\frac{\Lambda_{\mathrm{UV}}^d}{d/2}b_0 + \frac{\Lambda_{\mathrm{UV}}^{d-2}}{\frac{d}{2}-1}b_2 + \cdots + \log\frac{\Lambda_{\mathrm{UV}}^2}{\mu^2}b_d + \text{ finite }\right].
\end{aligned}
$$

(42)

These are relevant for the contributions from the ghost and the operator Y .

这些结果与鬼贡献和算符 Y 的贡献相关。

In our present case, the graviton contribution (25) is $p = 4$ and the ghost contributions are $p = 2$. So the one-loop part of our effective action is

在本文的情形中，引力子贡献 (25) 为 $p=4$ ，鬼贡献为 $p=2$ ，因此我们有效作用量的单圈部分为

$$
\begin{aligned}
\Gamma^{1-\text{ loop}} &= \frac{1}{2}\operatorname{Tr}\log\mathcal{H} - \operatorname{Tr}\log\Delta_{gh} - \frac{1}{2}\operatorname{Tr}\log Y \\
&= -\int \frac{d^4 x}{2(4\pi)^2}\sqrt{g}\Big[\Lambda_{\mathrm{UV}}^4\left(b_0(\mathcal{H}) - b_0(\Delta_{gh}) - \frac{1}{2}b_0(Y)\right) \\
&+\Lambda_{\mathrm{UV}}^2\left(2b_2(\mathcal{H}) - 2b_2(\Delta_{gh}) - b_2(Y)\right) \\
&+\log\frac{\Lambda_{\mathrm{UV}}^2}{\mu^2}\left(2b_4(\mathcal{H}) - 2b_4(\Delta_{gh}) - b_4(Y)\right)\Big].
\end{aligned}
\tag{43}
$$

Now we start the evaluation of these contributions.

现在我们开始计算这些贡献。

Contributions from $p = 4$ Minimal Operator

来自 $p=4$ 极小算符的贡献

First we have to evaluate the contribution from (25) which is $p = 4$ minimal operator. The coefficients in the heat kernel expansion for spin 2 are [13]:

首先我们需要计算式 (25) 中 $p = 4$ 极小算符的贡献。自旋 2 的热核展开系数如下 [13]:

$$b_0(\mathcal{H}) = \frac{\Gamma(d/4)}{2\Gamma(d/2)} \frac{d(d+1)}{2}, \tag{44}$$

$$b_2(\mathcal{H}) = \frac{\Gamma((d-2)/4)}{2\Gamma((d-2)/2)} \operatorname{tr}\left[\frac{\bar{R}}{6} + \frac{1}{2d} V_\mu^\mu\right], \tag{45}$$

$$\begin{aligned} b_4(\mathcal{H}) = \frac{\Gamma(d/4)}{2\Gamma((d-2)/2)} \operatorname{tr}\Big[& \frac{\hat{1}}{90}\bar{R}^2_{\rho\lambda\sigma\tau} - \frac{\hat{1}}{90}\bar{R}^2_{\rho\lambda} + \frac{\hat{1}}{36}\bar{R}^2 + \frac{1}{6}\Omega_{\rho\lambda}\Omega^{\rho\lambda} - \frac{2}{d-2}U \\ & - \frac{1}{6(d-2)}\left(2\bar{R}_{\rho\lambda}V^{\rho\lambda} - \bar{R}V^\rho{}_\rho\right) + \frac{1}{4(d^2-4)}\left(V^\rho{}_\rho V^\lambda{}_\lambda + 2V_{\rho\lambda}V^{\rho\lambda}\right) \\ & + \frac{1}{15}\Box\bar{R} + \frac{d+4}{6(d^2-4)}\Box V^\rho{}_\rho - \frac{2(d+1)}{3(d^2-4)}\bar{\nabla}^\rho\bar{\nabla}^\lambda V_{\rho\lambda}\Big], \end{aligned} \tag{46}$$

where $\hat{1}$ is the identity defined in (13) and $\Omega_{\rho\lambda}$ is the commutator of the covariant derivatives acting on the tensor $h^{\alpha\beta}$: $\Omega_{\rho\lambda} = \left[\bar{\nabla}_\rho, \bar{\nabla}_\lambda\right]$. The traces should be taken over the space of symmetric tensors with identity (13). These give $\operatorname{tr}(\hat{1}) = \frac{d(d+1)}{2}$, $\operatorname{tr}(\Omega_{\rho\lambda}\Omega^{\rho\lambda}) = -(d+2)\bar{R}^2_{\mu\nu\alpha\beta}$.

其中 $\hat{1}$ 是式 (13) 定义的单位矩阵，$\Omega_{\rho\lambda}$ 是作用在张量 $h^{\alpha\beta}$: $\Omega_{\rho\lambda} = \left[\bar{\nabla}_\rho, \bar{\nabla}_\lambda\right]$ 上的协变导数的对易子。迹应在带有单位矩阵 (13) 的对称张量空间上取，计算结果为 $\operatorname{tr}(\hat{1}) = \frac{d(d+1)}{2}$, $\operatorname{tr}(\Omega_{\rho\lambda}\Omega^{\rho\lambda}) = -(d+2)\bar{R}^2_{\mu\nu\alpha\beta}$.

We need more explicit formulae for the traces. Because these are very complicated for general dimensions, we give the results for $d = 4$ and omit total derivative terms. We find

我们需要更明确的迹公式，由于这些表达式对任意维度都十分复杂，下文仅给出 $d = 4$ 的结果并省略全导数项，我们得到

$$\operatorname{tr} U = \delta^{\mu\nu,\alpha\beta} U_{\mu\nu,\alpha\beta} = A_1\bar{R}^2_{\mu\nu\rho\lambda} + A_2\bar{R}^2_{\mu\nu} + A_3\bar{R}^2 - A_4\frac{\bar{R}}{\kappa^2} - A_5\frac{\Lambda}{\kappa^2}, \tag{47}$$

where

其中

$$A_1 = 3,\ A_2 = \frac{8}{3} + \frac{4\lambda}{\xi},\ A_3 = \frac{1}{3} + \frac{2\lambda}{\xi},\ A_4 = 3\lambda,\ A_5 = \frac{2}{9}(84\lambda - \xi), \tag{48}$$

and

且

$$\operatorname{tr}\left(V_\rho^\rho \bar{R}\right) = B_1\bar{R}^2 - B_2\frac{\bar{R}}{\kappa^2}, \tag{49}$$

where

其中

$$B_1 = -\frac{68}{3} + \frac{32\lambda}{\xi},\ B_2 = -20\lambda + \frac{2\xi}{3}. \tag{50}$$

Next,

接下来,

$$\mathrm{tr}\left(V^{\rho\lambda}\bar{R}_{\rho\lambda}\right) = C_1\bar{R}_{\mu\nu}^2 + C_2\bar{R}^2 - C_3\frac{\bar{R}}{\kappa^2}, \tag{51}$$

where

其中

$$C_1 = \frac{8}{3} - \frac{8\lambda}{\xi},\ C_2 = -\frac{19}{3} + \frac{10\lambda}{\xi},\ C_3 = -5\lambda + \frac{\xi}{6}. \tag{52}$$

Finally,

最后,

$$\frac{1}{48}\,\mathrm{tr}\left(V_\rho^\rho V_\lambda^\lambda\right) + \frac{1}{24}\,\mathrm{tr}\left(V_{\rho_\lambda}V^{\rho\lambda}\right) = D_1\bar{R}_{\mu\nu\rho\lambda}^2 + D_2\bar{R}_{\mu\nu}^2 + D_3\bar{R}^2 - D_4\frac{\bar{R}}{\kappa^2} + D_5\frac{1}{\kappa^4}, \tag{53}$$

where

其中

$$\begin{gathered} D_1 = 6,\ D_2 = \frac{2\left(18\lambda^2 + 6\lambda\xi + 113\xi^2\right)}{27\xi^2},\ D_3 = \frac{576\lambda^2 - 240\lambda\xi - 47\xi^2}{54\xi^2}, \\ D_4 = \frac{-180\lambda^2 + 30\lambda\xi + \xi^2}{18\xi},\ D_5 = \frac{180\lambda^2 + \xi^2}{72}. \end{gathered} \tag{54}$$

Contribution from the $p = 2$ Nonminimal Vector Operator

来自 $p = 2$ 非最小矢量算符的贡献

To find the contribution from the ghost operator, we need the contribution from $p = 2$ nonminimal vector operator. The general form of the $p = 2$ nonminimal operator is

为了求得鬼算子的贡献，我们需要得到 $p = 2$ 非最小矢量算子的贡献。$p = 2$ 非最小算子的一般形式为

$$\Delta = -\bar{g}^{\mu\upsilon}\Box + a\overline{\nabla}^{\mu}\overline{\nabla}^{\upsilon} + X^{\mu\upsilon}. \tag{55}$$

The coefficients in the heat kernel expansion (40) have been calculated in [14]. For the general case, we have

热核展开 (40) 中的系数已在文献 [14] 中算出。对于一般情况，我们有

$$b_0 = (1-a)^{-d/2} + d - 1 \tag{56}$$

$$b_2 = \left(\frac{d^2-d-6}{6d} + (1-a)^{-d/2}\frac{6+(1-a)d}{6d}\right)\bar{R} + \frac{1-d-(1-a)^{-d/2}}{d}X,$$

(57)

$$b_4 = \frac{d-16+(1-a)^{2-d/2}}{180}\bar{R}^2_{\mu\nu\rho\lambda}$$

$$+\frac{1}{180ad\left(d^2-4\right)}\Big[-360d-\left(d^4-d^3+116d^2-296d-360\right)a+(1-a)^{-d/2}[-360a$$

$$+4\left(90-74a+28a^2+a^3\right)d-60a(1-a)d^2-(1-a)^2ad^3]\Big]\bar{R}^2_{\mu\nu}$$

$$+\frac{1}{72ad\left(d^2-4\right)}\big[72(2-a)+\left(16d-16d^2-d^3+d^4\right)a$$

$$+(1-a)^{-d/2}\Big[-72(2-a)-4a\left(a^2+4a-14\right)d+12a(1-a)d^2+a(1-a)^2d^3\Big]\Big]\bar{R}^2$$

$$+\frac{1}{6ad\left(d^2-4\right)}\Big[\Big\{-24+12a+ad^2-ad^3+(1-a)^{-d/2}[24-12a$$

$$+2a(a-4)d-a(1-a)d^2]\Big\}\bar{R}X+2\big\{-12a+4(3-2a)d+5ad^2$$

$$+(1-a)^{-d/2}\left[12a-2\left(6-4a+a^2\right)d+a(1-a)d^2\right]\Big]\bar{R}_{\mu\nu}X^{\mu\nu}$$

$$+3\Big\{4-2a+ad+(1-a)^{-d/2}(-4+2a+ad)\Big\}X^2+3\big\{4a-2(2+a)d-2ad^2$$

$$+ad^3+(1-a)^{-d/2}[-4a+2(2-a)d\}\Big]X_{\mu\upsilon}X^{\mu\upsilon}] + \text{ (total derivative terms)},$$

(58)

where $X = X^{\mu}_{\mu}$.

其中 $X = X^{\mu}_{\mu}$ 。

Contribution from the Ghost Operator Δ_{gh}

鬼算符的贡献 Δ_{gh}

For the ghost operator Δ_{gh} in (19), we see that we have $a=-1-\frac{4\alpha+\beta}{2(\gamma-\alpha)}\equiv\sigma_g$ and $X_{\mu\upsilon}=-\bar{R}_{\mu\upsilon}$ in (55). The above formulae give, for $d=4$,

对于式 (19) 中的鬼算符 Δ_{gh} ，可见式 (55) 中存在 $a=-1-\frac{4\alpha+\beta}{2(\gamma-\alpha)}\equiv\sigma_g$ 和 $X_{\mu\upsilon}=-\bar{R}_{\mu\upsilon}$ 。上述公式给出，当 $d=4$ 时，

$$b_0\left(\Delta_{gh}\right)=\left(1-\sigma_g\right)^{-2}+3, \tag{59}$$

$$b_2\left(\Delta_{gh}\right)=\frac{6\sigma_g^2-13\sigma_g+10}{6\left(1-\sigma_g\right)^2}\bar{R}, \tag{60}$$

$$b_4\left(\Delta_{gh}\right)=\frac{-11}{180}\bar{R}_{\mu\nu\rho\lambda}^2-\frac{2\sigma_g^2+26\sigma_g-43}{90\left(1-\sigma_g\right)^2}\bar{R}_{\mu\nu}^2+\frac{5\sigma_g^2-10\sigma_g+8}{36\left(1-\sigma_g\right)^2}\bar{R}^2. \tag{61}$$

Contribution from the Additional Ghost Operator Y

额外鬼算子 Y 的贡献

For the operator Y in (19), we first note that $\left[\bar{\nabla}_\mu,\bar{\nabla}_\upsilon\right]\chi^\upsilon=-\bar{R}_{\mu\upsilon}\chi^\upsilon$. Using this relation, we find that we have $a=1+2\frac{\alpha-\gamma}{\beta+4\gamma}\equiv\sigma_Y$ and $X_{\mu\upsilon}=\bar{R}_{\mu\upsilon}$ in (55), and the above formulae give, for $d=4$,

对于式 (19) 中的算子 Y ，我们首先注意到 $\left[\bar{\nabla}_\mu,\bar{\nabla}_\upsilon\right]\chi^\upsilon=-\bar{R}_{\mu\upsilon}\chi^\upsilon$ 。利用该关系，我们可知式 (55) 中存在 $a=1+2\frac{\alpha-\gamma}{\beta+4\gamma}\equiv\sigma_Y$ 和 $X_{\mu\upsilon}=\bar{R}_{\mu\upsilon}$ ，对于 $d=4$ ，上述公式给出:

$$b_0(Y)=\left(1-\sigma_Y\right)^{-2}+3, \tag{62}$$

$$b_2(Y)=\frac{3\sigma_Y-2}{6\left(1-\sigma_Y\right)}\bar{R}, \tag{63}$$

$$b_4(Y)=\frac{-11}{180}\bar{R}_{\mu\upsilon\rho\lambda}^2+\frac{43}{90}\bar{R}_{\mu\upsilon}^2-\frac{1}{9}\bar{R}^2. \tag{64}$$

One-Loop Divergences and Asymptotic Freedom

单圈发散与渐近自由

We are now ready to calculate the divergences in the quadratic curvature theory on the general backgrounds. For this purpose, we need to know the heat kernel coefficients. In this section, we restrict the dimension of our spacetime to four.

我们现在已经可以计算一般背景下二次曲率理论中的发散。为此我们需要知道热核系数。本节我们将时空维度限定为四维。

Putting the results for $\mathcal{H}$ into Eq. (46), we get

将 $\mathcal{H}$ 的结果代入式 (46)，我们得到

$$b_0(\mathcal{H}) = 5, \tag{65}$$

$$b_2(\mathcal{H}) = \frac{\sqrt{\pi}}{2}\left[\frac{5\bar{R}}{3} + \frac{1}{8}\left(B_1\bar{R} - B_2\frac{1}{\kappa^2}\right)\right], \tag{66}$$

$$\begin{aligned} b_4(\mathcal{H}) = \frac{1}{2}\Big[&\bar{R}^2_{\mu\nu\rho\lambda}\left(-\frac{8}{9} - A_1 + D_1\right) - \bar{R}^2_{\mu\nu}\left(\frac{1}{9} + A_2 + \frac{1}{6}C_1 - D_2\right) \\ &+\bar{R}^2\left(\frac{5}{18} - A_3 + \frac{1}{12}B_1 - \frac{1}{6}C_2 + D_3\right) + \frac{1}{\kappa^2}\bar{R}\left(A_4 - \frac{1}{12}B_2 + \frac{1}{6}C_3 - D_4\right) \\ &+\frac{1}{\kappa^2}\left(\Lambda A_5 + \frac{1}{\kappa^2}D_5\right)\Big], \end{aligned} \tag{67}$$

where the constants A_i, B_i, C_i , and D_i are given in (48)-(54).

其中常数 A_i, B_i, C_i 和 D_i 由式 (48)-(54) 给出。

Collecting other contributions from ghosts and Y , we finally get

汇总来自鬼场和 Y 的其他贡献后，我们最终得到

$$\begin{aligned} \Gamma^{1-\text{ loop}} = -\int \frac{d^4x}{2(4\pi)^2}\Bigg[\Bigg\{&\frac{133}{20}C^2 + \left(10\frac{\lambda^2}{\xi^2} - 5\frac{\lambda}{\xi} + \frac{5}{36}\right)\bar{R}^2 - \frac{196}{45}E \\ &+\frac{(30\lambda-\xi)(4\lambda+\xi)}{12\xi\kappa^2}\bar{R} + 2\frac{84\lambda-\xi}{9\kappa^2}\Lambda + \frac{180\lambda^2+\xi^2}{72\kappa^4}\Bigg\}\log\frac{\Lambda_{\text{UV}}^2}{\mu^2} \\ &-\Bigg\{\frac{144\lambda^3 - 24\left(7+6\sqrt{\pi}\right)\lambda^2\xi + 2\left(59+45\sqrt{\pi}\right)\lambda\xi^2 - \left(29+14\sqrt{\pi}\right)\xi^3}{12(3\lambda-\xi)\xi^2}\bar{R} \\ &+\frac{\sqrt{\pi}(30\lambda-\xi)}{12\kappa^2}\Bigg\}\Lambda_{\text{UV}}^2 \\ &-\frac{2592\lambda^4 - 3456\lambda^3\xi + 3024\lambda^2\xi^2 - 1248\lambda\xi^3 + 257\xi^4}{72\xi^2(3\lambda-\xi)^2}\Lambda_{\text{UV}}^4\Bigg]. \end{aligned}$$

(68)

Note that the coefficient of the Euler term E is independent of any coupling, in particular of its coupling ρ . This is to be expected, because the Euler term is a topological term and is a total derivative itself, and as such it does not contribute to the Hessian and therefore to quantum effects [21]. Thus it is a universal result that it is independent of the coupling ρ whatever the approximation is (beyond one loop).

注意欧拉项 E 的系数不依赖于任何耦合，尤其不依赖于它自身的耦合 ρ 。这符合预期: 因为欧拉项是拓扑项，本身就是全导数，因此它不会对黑塞矩阵产生贡献，进而也不会对量子效应产生贡献 [21]。因此，无论何种近似 (单圈阶以上)，该系数都不依赖于耦合 ρ 是一个普适结论。

The correspondence between the cutoff and the dimensional regularization is $\log \frac{\Lambda_{\mathrm{UV}}^2}{\mu^2} \leftrightarrow \frac{2}{4-d}$ and one can try to compare the results with the existing literature (see, e.g., [16, 22]).

截断正则化与维数正规化之间的对应关系为 $\log \frac{\Lambda_{\mathrm{UV}}^2}{\mu^2} \leftrightarrow \frac{2}{4-d}$ ，我们可以将结果与现有文献比较 (例如参见 [16, 22])。

Note also that there are strange terms with coefficients $\sqrt{\pi}$ in the quadratic divergences. Indeed, if we consider the minimal operators $F_1 = \Box + P_1$ and $F_2 = \Box + P_2$ and consider the quadratic divergences of $\operatorname{Tr}\log(F_1F_2)$, it appears that we get such factor of $\sqrt{\pi}$ from the formula b_2 in (66), whereas if we write it as $\operatorname{Tr}\log F_1 + \operatorname{Tr}\log F_2$, we do not get $\sqrt{\pi}$ as is clear from (57) for the same quantity. This clearly indicates that the coefficients of the quadratic divergences depend on how we calculate. Thus the power divergences are not universal and do not lead to physical effects like renormalization group scaling. The above results are just those obtained by the naive application of the formulae but should not be taken seriously.

还需注意，二次发散中存在系数为 $\sqrt{\pi}$ 的反常项。事实上，如果我们考虑最小算符 $F_1 = \Box + P_1$ 和 $F_2 = \Box + P_2$ ，并研究 $\operatorname{Tr}\log(F_1F_2)$ 的二次发散，我们会从式 (66) 中的公式 b_2 得到 $\sqrt{\pi}$ 这样的因子；但如果我们将其写为 $\operatorname{Tr}\log F_1 + \operatorname{Tr}\log F_2$ ，对照同一量的式 (57) 可以发现，我们得不到 $\sqrt{\pi}$ 。这清楚表明二次发散的系数依赖于我们的计算方式。因此幂次发散不具有普适性，也不会产生类似重整化群标度的物理效应。上述结果只是公式朴素应用得到的结论，并不可靠。

On the other hand, the logarithmic divergences are universal. From the coefficients, we can determine the beta functions for the dimensionless couplings: recall that together with the bare terms, the coefficients of C^2 should give the renormalized coupling

另一方面，对数发散是普适的。我们可以从系数得到无量纲耦合的 β 函数: 回想一下，结合裸项，C^2 的系数应给出重整化耦合

$$\frac{1}{2\lambda_R} = \frac{1}{2\lambda_B} - \frac{133}{(4\pi)^2 40} \log \frac{\Lambda_{\mathrm{UV}}^2}{\mu^2}. \tag{69}$$

Since the bare coupling λ_B does not depend on the renormalization scale μ , differentiation of the expression with respect to $\log\mu$ gives

由于裸耦合 λ_B 不依赖于重整化标度 μ ，对表达式关于 $\log\mu$ 求导可得

$$-\frac{1}{2\lambda_R^2}\mu\frac{d\lambda_R}{d\mu} = \frac{133}{(4\pi)^2 20}. \tag{70}$$

Omitting the subscript R , this gives the beta function of the coupling λ :

省去下标 R 后，我们得到耦合 λ 的 β 函数:

$$\mu \frac{d\lambda}{d\mu} = \beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2. \tag{71}$$

Similarly we find the beta functions for other dimensionless couplings:

同理我们可得其他无量纲耦合的 β 函数:

$$\beta_\xi = -\frac{1}{(4\pi)^2}\left(10\lambda^2 - 5\lambda\xi + \frac{5}{36}\xi^2\right)$$

$$\beta_\rho = -\frac{1}{(4\pi)^2}\frac{196}{45}\rho^2 \tag{72}$$

The first equation (71) tells us that the coupling λ goes to zero from positive λ . Similarly the other couplings also go to zero. This is known as asymptotic freedom $[9, 10, 16-20]$. However it has recently been discovered that there are fixed point at finite values for these couplings [21] in addition to these Gaussian fixed points. See, however [23].

第一个方程 (71) 告诉我们，耦合 λ 从正值 λ 趋近于零。其他耦合也同样趋近于零。这就是我们所知的渐近自由 $[9, 10, 16-20]$ 。但最近研究发现，除了这些高斯不动点外，这些耦合还在有限值处存在不动点 [21]，另见文献 [23]。

Divergences for $f\left(R, R_{\mu\nu}^2\right)$ Gravity

$f\left(R, R_{\mu\nu}^2\right)$ 引力的发散

As an interesting case of higher-derivative gravity, here we present the one-loop divergences for $f\left(R, R_{\mu\nu}\right)$ gravity on the Einstein space [24]:

作为高阶导数引力的一个有趣实例，我们在此给出爱因斯坦空间上 $f\left(R, R_{\mu\nu}\right)$ 引力的单圈发散 [24]:

$$\bar{R}_{\mu\nu} = \frac{\bar{R}}{d}\bar{g}_{\mu\nu}. \tag{73}$$

Moreover we consider a general parametrization of the fluctuations

此外我们考虑涨落的一般参数化

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \tag{74}$$

where the fluctuation is expanded:

其中涨落展开为:

$$\delta g_{\mu\nu} = \delta g_{\mu\nu}^{(1)} + \delta g_{\mu\nu}^{(2)} + \delta g_{\mu\nu}^{(3)} + \dots, \tag{75}$$

where $\delta g^{(n)}_{\mu\nu}$ contains n powers of $h_{\mu\nu}$. We will parametrize the first two terms of the expansion as follows:

其中 $\delta g^{(n)}_{\mu\nu}$ 包含 n 次 $h_{\mu\nu}$ 幂次。我们将展开的前两项按如下方式参数化:

$$\delta g^{(1)}_{\mu\nu} = h_{\mu\nu} + m\bar{g}_{\mu\nu}h$$

$$\delta g^{(2)}_{\mu\upsilon} = \omega h_{\mu\rho}h^{\rho}{}_{\upsilon} + mhh_{\mu\upsilon} + m\left(\omega - \frac{1}{2}\right)\bar{g}_{\mu\upsilon}h^{\alpha\beta}h_{\alpha\beta} + \frac{1}{2}m^2\bar{g}_{\mu\upsilon}h^2. \tag{76}$$

It is convenient to use York decomposition

使用约克分解更为方便

$$h_{\mu\upsilon} = h^{TT}_{\mu\upsilon} + \overline{\nabla}_{\mu}\xi_{\upsilon} + \overline{\nabla}_{\upsilon}\xi_{\mu} + \overline{\nabla}_{\mu}\overline{\nabla}_{\upsilon}\sigma - \frac{1}{d}\bar{g}_{\mu\upsilon}\overline{\nabla}^{2}\sigma + \frac{1}{d}\bar{g}_{\mu\upsilon}h, \tag{77}$$

where

其中

$$\overline{\nabla}^{\mu}h^{TT}_{\mu\upsilon} = 0;\ \bar{g}^{\mu\upsilon}h^{TT}_{\mu\upsilon} = 0;\ \overline{\nabla}_{\mu}\xi^{\mu} = 0.$$

We then find that the Hessian is

我们随后得到黑塞矩阵为

$$S^{(2)} = \int d^dx\sqrt{\bar{g}}\left[h^{TT}_{\mu\upsilon}H^{TT}h^{TT\mu\upsilon} + \xi_{\mu}H^{\xi\xi}\xi^{\mu} + \sigma H^{\sigma\sigma}\sigma + \sigma H^{\sigma h}h + hH^{h\sigma}\sigma\right.$$
$$\left. + hH^{hh}h\right] \tag{78}$$

where

其中

$$H^{TT} = \frac{1}{4}\left[\left\{\bar{f}_X\left(\Delta_{L2} - \frac{4\bar{R}}{d}\right) - \bar{f}_R\right\}\left(\Delta_{L2} - \frac{2\bar{R}}{d}\right) - (1-2\omega)(1+md)\widetilde{E}\right],$$

(79)

$$H^{\xi\xi} = -\frac{(1-2\omega)(1+md)}{2}\left(\Delta_{L1} - \frac{2\bar{R}}{d}\right)\widetilde{E}, \tag{80}$$

$$H^{\sigma\sigma} = \frac{1}{2}\left(\frac{d-1}{d}\right)^2\left[P\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right) + Q\Delta_{L0}\right.$$
$$\left. - \frac{d(1-2\omega)(1+md)}{2(d-1)}\widetilde{E}\right]\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1}\right), \tag{81}$$

$$H^{\sigma h}=\left(\frac{d-1}{d}\right)^2\frac{1+md}{2}\left[P\left(\Delta_{L0}-\frac{\bar{R}}{d-1}\right)+Q\right]\Delta_{L0}\left(\Delta_{L0}-\frac{\bar{R}}{d-1}\right),$$
(82)

$$H^{hh}=\left(\frac{d-1}{d}\right)^2\frac{(1+md)^2}{2}\left[P\left(\Delta_{L0}-\frac{\bar{R}}{d-1}\right)^2+Q\left(\Delta_{L0}-\frac{\bar{R}}{d-1}\right)\right.$$
$$\left.+\frac{d\left[(1+md)\,d-2\,(1-2\omega)\right]}{4(d-1)^2\,(1+md)}\widetilde{E}\right],\tag{83}$$

where Δ_{L2}, Δ_{L1} , and Δ_{L0} are the Lichnerowicz Laplacians defined as

其中 Δ_{L2}, Δ_{L1} 和 Δ_{L0} 是利希纳罗维奇拉普拉斯算子，定义为

$$\Delta_{L2}T_{\mu\nu}=-\overline{\nabla}^2T_{\mu\nu}+\bar{R}_\mu{}^\rho T_{\rho\nu}+\bar{R}_\upsilon{}^\rho T_{\mu\rho}-\bar{R}_{\mu\rho\nu\sigma}T^{\rho\sigma}-\bar{R}_{\mu\rho\nu\sigma}T^{\sigma\rho},$$

$$\Delta_{L1}V_\mu=-\overline{\nabla}^2V_\mu+\bar{R}_\mu{}^\rho V_\rho$$

$$\Delta_{L0}S=-\overline{\nabla}^2S\tag{84}$$

and the subscripts on f denote derivatives with respect to its arguments:

f 的下标表示对其自变量的导数:

$$f_R=\frac{\partial f}{\partial\bar{R}},\ f_X=\frac{\partial f}{\partial X},\ f_{RR}=\frac{\partial^2 f}{\partial\bar{R}^2},\ f_{RX}=\frac{\partial^2 f}{\partial\bar{R}\partial X},\ f_{XX}=\frac{\partial^2 f}{\partial X^2},\tag{85}$$

with

其中

$$X\equiv\bar{R}_{\mu\nu}^2.\tag{86}$$

We have also used the shorthands

我们还使用了简写

$$P=\bar{f}_{RR}+\frac{4}{d^2}\bar{R}^2\bar{f}_{XX}+4\bar{R}\bar{f}_{RX}+\frac{d}{2\,(d-1)}\bar{f}_X\tag{87}$$

$$Q=\frac{d-2}{2\,(d-1)}\bar{f}_R+\frac{3d^2-10d+8}{2d(d-1)^2}\bar{R}\bar{f}_X\tag{88}$$

and

且

$$\widetilde{E} \equiv \bar{f} - \frac{2}{d}\bar{R}\bar{f}_R - \frac{4\bar{R}^2}{d^2}\bar{f}_X = 0 \tag{89}$$

is the field equation evaluated on the Einstein space (73).

是爱因斯坦空间 (73) 上满足的场方程。

Our gauge fixing is

我们的规范固定为

$$S_{GF} = \frac{1}{2a}\int d^d x\sqrt{\bar{g}}\bar{g}^{\mu\upsilon}F_\mu F_\upsilon, \tag{90}$$

with

其中

$$F_\mu = \bar{\bar{\nabla}}_\alpha h^\alpha{}_\mu - \frac{\bar{b}+1}{d}\bar{\bar{\nabla}}_\mu h, \tag{91}$$

and a and $\bar{b}$ are gauge parameters. This can be rewritten as

且 a 和 $\bar{b}$ 为规范参数。上式可改写为

$$\begin{aligned} S_{GF} = -\frac{1}{2a}\int d^d x\sqrt{\bar{g}}\Bigg[\xi_\mu\left(\Delta_{L1} - \frac{2\bar{R}}{d}\right)^2\xi^\mu \\ +\frac{(d-1-b)^2}{d^2}\chi\Delta_{L0}\left(\Delta_{L0} - \frac{\bar{R}}{d-1-b}\right)^2\chi\Bigg], \end{aligned} \tag{92}$$

in terms of the new field

用新场表示为

$$\chi = \frac{(d-1)\,\Delta_{L0} - \bar{R}}{(d-1-b)\,\Delta_{L0} - \bar{R}}\sigma + \frac{b\,(1+dm)}{(d-1-b)\,\Delta_{L0} - \bar{R}}h, \tag{93}$$

where $b = \bar{b}/\,(1+md)$.

其中 $b = \bar{b}/\,(1+md)$ 。

The ghost action contains a nonminimal operator

鬼作用量包含一个非最小算符

$$S_{gh} = i\int d^d x\sqrt{\bar{g}}\bar{C}^\mu\left(\delta^\upsilon_\mu\overline{\nabla}^2 + \left(1 - 2\frac{b+1}{d}\right)\overline{\nabla}_\mu\overline{\nabla}^\upsilon + \bar{R}_\mu{}^\upsilon\right)C_\upsilon. \tag{94}$$

We can rewrite this as

我们可将其改写为

$$S_{gh} = i \int d^d x \sqrt{\bar{g}} \Big[\bar{C}^{T\mu} \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right) C^T_\mu$$

$$+2 \frac{d-1-b}{d} \bar{C}'^L \left(\Delta_{L0} - \frac{\bar{R}}{d-1-b} \right) C'^L \Big] \tag{95}$$

in terms of the transverse and longitudinal parts of the ghost field:

用鬼场的横向部分与纵向部分表示:

$$C_v = C^T_v + \overline{\nabla}_v C^L = C^T_v + \overline{\nabla}_v \frac{1}{\sqrt{\Delta_{L0}}} C'^L \tag{96}$$

and the same for $\bar{C}$. This change of variables has unit Jacobian.

且对 $\bar{C}$ 也做同样处理。该变量替换的雅可比行列式为单位阵。

$$S_{gh} = i \int d^d x \sqrt{\bar{g}} \Big[\bar{C}^{T\mu} \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right) C^T_\mu$$

$$+2 \frac{d-1-b}{d} \bar{C}'^L \left(\Delta_{L0} - \frac{\bar{R}}{d-1-b} \right) C'^L \Big] . \tag{97}$$

Unless we (1) set $\omega = \frac{1}{2}$ or (2) put $m = -\frac{1}{d}$ or (3) go on shell, the effective action is gauge dependent. If we impose the on-shell condition (89). the effective action is gauge independent and is independent of ω and m . In this case, we find

除非我们 (1) 取 $\omega = \frac{1}{2}$ ，或 (2) 取 $m = -\frac{1}{d}$ ，或 (3) 满足在壳条件，否则有效作用量依赖于规范选择。若我们施加在壳条件 (89)，有效作用量就是规范无关的，且与 ω 和 m 无关。在此情况下我们得到

$$\Gamma = \frac{1}{2} \operatorname{Tr} \log \left(\Delta_{L2} - \frac{4\bar{R}}{d} - \frac{\bar{f}_R}{\bar{f}_X} \right) + \frac{1}{2} \operatorname{Tr} \log \left(\Delta_{L2} - \frac{2\bar{R}}{d} \right)$$

$$+\frac{1}{2} \operatorname{Tr} \log \left(\Delta_{L0} - \frac{\bar{R}}{d-1} + \frac{Q}{P} \right) - \frac{1}{2} \operatorname{Tr} \log \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right). \tag{98}$$

If $\bar{f}_X = 0$, the first contribution is absent.

若取 $\bar{f}_X = 0$ ，则第一项贡献为零。

The divergent part of the effective action can be computed by the heat kernel methods. On an Einstein background in four dimensions, with the help of the heat kernel coefficients for the Lichnerowicz operator summarized in Appendix "Heat Kernel Coefficients for $p = 2$ Minimal Operator," the logarithmically divergent part is found to be [24]

有效作用量的发散部分可通过热核方法计算。对于四维空间中的爱因斯坦背景，借助附录“$p = 2$ 最小算符的热核系数”中汇总的里奇纳罗维奇算符的热核系数，我们得到对数发散部分为 [24]

$$\Gamma_{\log}(\bar{g}) = \frac{1}{720(4\pi)^2}\int d^4x\sqrt{\bar{g}}\log\left(\frac{\Lambda_{\mathrm{UV}}^2}{\mu^2}\right)\Bigg[-826\bar{R}_{\mu\nu\rho\sigma}^2 + 509\bar{R}^2 - \frac{300\bar{R}\bar{f}_R}{\bar{f}_X}$$
$$-\frac{900\bar{f}_R^2}{\bar{f}_X^2} + \frac{240\bar{R}\left(3\bar{f}_R + 2\bar{R}\bar{f}_X\right)}{8\bar{f}_X + 12\bar{f}_{RR} + 48\bar{R}\bar{f}_{RX} + 3\bar{R}^2\bar{f}_{XX}}$$
$$-\frac{320\left(3\bar{f}_R + 2\bar{R}\bar{f}_X\right)^2}{\left(8\bar{f}_X + 12\bar{f}_{RR} + 48\bar{R}\bar{f}_{RX} + 3\bar{R}^2\bar{f}_{XX}\right)^2}\Bigg], \tag{99}$$

where Λ_{UV} stands for a cutoff and we introduced a reference mass scale μ .

其中 Λ_{UV} 代表截断，我们引入了参考质量标度 μ 。

For the choice

对于选择

$$f(R,X) = \alpha\bar{R}^2 + \beta X, \tag{100}$$

it reduces to

它可约化为

$$\Gamma_{\log}(\bar{g}) = \frac{1}{(4\pi)^2}\int d^4x\sqrt{\bar{g}}\log\left(\frac{\Lambda_{\mathrm{UV}}^2}{\mu^2}\right)\left[-\frac{413}{360}\bar{R}_{\mu\nu\rho\sigma}^2 - \frac{1200\alpha^2 + 200\alpha\beta - 183\beta^2}{240\beta^2}\bar{R}^2\right],$$

(101)

which is the standard universal result in higher-derivative gravity.

这就是高导数引力中的标准普适结果。

On the other hand if we put

另一方面，若我们取

$$f(R,X) = f(R), \tag{102}$$

we obtain

我们得到

$$\Gamma_{\log}(\bar{g}) = \frac{1}{(4\pi)^2}\int d^4x\sqrt{\bar{g}}\log\left(\frac{\Lambda_{\mathrm{UV}}^2}{\mu^2}\right)\left[-\frac{71}{120}\bar{R}_{\mu\nu\rho\sigma}^2 + \frac{433}{1440}\bar{R}^2 + \frac{\bar{f}_R\bar{R}}{12\bar{f}_{RR}} - \frac{\bar{f}_R^2}{36\bar{f}_{RR}^2}\right],$$

(103)

which agrees with the results of [25, 26] .

这与 [25, 26] 的结果一致。

Cross-References

交叉引用

Perturbative Approaches to Nonperturbative Quantum Gravity

非微扰量子引力的微扰方法

- The Functional Renormalization Group in Quantum Gravity

- 量子引力中的泛函重整化群

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Appendix

附录

Expansion of Curvatures up to Second Order

曲率的二阶展开

Here we summarize our conventions and formulae necessary in the text. We give these such that they are valid for any dimension d .

我们在此汇总正文所需的约定与公式，给出的结果对任意维数 d 都成立。

Our signature of the metric is $(-,+,\cdots+)$ and the curvature tensors are given as

我们的度规符号差为 $(-,+,\cdots+)$ ，曲率张量定义如下

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\beta\nu} - \Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\beta\mu},$$

$$R_{\mu\upsilon} = \bar{R}^{\alpha}{}_{\mu\alpha\upsilon} \tag{104}$$

The backgrounds are denoted with overbar. Expansion around the background gives

背景场带有上划线标记，围绕背景展开可得

$$\Gamma^{\alpha}_{\mu\nu} = \overline{\Gamma}^{\alpha}_{\mu\nu} + \Gamma^{\alpha(1)}_{\mu\nu} + \Gamma^{\alpha(2)}_{\mu\nu}, \tag{105}$$

where

其中

$$\Gamma^{\alpha(1)}_{\mu\nu} = \frac{1}{2}\left(\overline{\nabla}_{\nu} h^{\alpha}{}_{\mu} + \overline{\nabla}_{\mu} h^{\alpha}{}_{\nu} - \overline{\nabla}^{\alpha} h_{\mu\nu}\right), \tag{106}$$

$$\Gamma^{\alpha(2)}_{\mu\upsilon} = -\frac{1}{2} h^{\alpha\beta}\left(\overline{\nabla}_{\upsilon} h_{\mu\beta} + \overline{\nabla}_{\mu} h_{\upsilon\beta} - \overline{\nabla}_{\beta} h_{\mu\upsilon}\right). \tag{107}$$

Note that

注意

$$\sqrt{-g} = \sqrt{-\bar{g}}\left[1 + \frac{1}{2}h + \frac{1}{8}\left(h^2 - 2h^2_{\mu\upsilon}\right) + O\left(h^3\right)\right], \tag{108}$$

where $h = h_{\mu}{}^{\mu}$. We find, to the second order,

其中 $h = h_{\mu}{}^{\mu}$ 。我们得到，到二阶为止，

$$R^{\mu}{}_{\nu\alpha\beta} = \bar{R}^{\mu}{}_{\nu\alpha\beta} + \bar{R}^{\mu(1)}_{\nu\alpha\beta} + \bar{R}^{\mu(2)}_{\nu\alpha\beta},$$

$$R^{\mu(1)(1)}_{\nu\alpha\beta} = \frac{1}{2}\left(\overline{\nabla}_{\alpha}\overline{\nabla}_{\upsilon} h^{\mu}_{\beta} - \overline{\nabla}_{\alpha}\overline{\nabla}^{\mu} h_{\upsilon\beta} - \overline{\nabla}_{\beta}\overline{\nabla}_{\upsilon} h^{\mu}_{\alpha} + \overline{\nabla}_{\beta}\overline{\nabla}^{\mu} h_{\upsilon\alpha}\right)$$

$$+\frac{1}{2}\bar{R}_{\upsilon\gamma\alpha\beta} h^{\mu\gamma} + \frac{1}{2}\bar{R}^{\mu}{}_{\gamma\alpha\beta} h^{\gamma}_{\upsilon}, \tag{109}$$

$$R^{\mu(2)}_{\nu\alpha\beta} = -\frac{1}{2} h^{\mu\gamma}\overline{\nabla}_{\alpha}\left(\overline{\nabla}_{\beta} h_{\nu\gamma} + \overline{\nabla}_{\nu} h_{\beta\gamma} - \overline{\nabla}_{\gamma} h_{\nu\beta}\right)$$

$$-\frac{1}{4}\overline{\nabla}_{\alpha} h^{\mu\gamma}\left(\overline{\nabla}_{\beta} h_{\upsilon\gamma} + \overline{\nabla}_{\upsilon} h_{\beta\gamma} - \overline{\nabla}_{\gamma} h_{\upsilon\beta}\right)$$

$$+\frac{1}{4}\overline{\nabla}_{\gamma} h^{\mu}_{\alpha}\left(\overline{\nabla}_{\beta} h^{\gamma}_{\upsilon} + \overline{\nabla}_{\upsilon} h^{\gamma}_{\beta} - \overline{\nabla}^{\gamma} h_{\upsilon\beta}\right)$$

$$-\frac{1}{4}\overline{\nabla}^{\mu} h_{\alpha\gamma}\left(\overline{\nabla}_{\beta} h^{\gamma}_{\upsilon} + \overline{\nabla}_{\upsilon} h^{\gamma}_{\beta} - \overline{\nabla}^{\gamma} h_{\upsilon\beta}\right) - (\alpha \leftrightarrow \beta). \tag{110}$$

Similarly

同理

$$R^{(1)}_{\mu\nu} = -\frac{1}{2}\left(\overline{\nabla}_{\mu}\overline{\nabla}_{\nu} h - \overline{\nabla}_{\mu} h_{\nu} - \overline{\nabla}_{\nu} h_{\mu} + \Box h_{\mu\nu}\right) - \bar{R}_{\alpha\mu\beta\nu} h^{\alpha\beta} + \frac{1}{2}\bar{R}_{\mu\alpha} h^{\alpha}_{\nu} + \frac{1}{2}\bar{R}_{\nu\alpha} h^{\alpha}_{\mu},$$

$$R_{\mu\upsilon}^{(2)} = \frac{1}{2}\overline{\nabla}_\mu\left(h^{\alpha\beta}\overline{\nabla}_\upsilon h_{\alpha\beta}\right) - \frac{1}{2}\overline{\nabla}_\alpha\left\{h^{\alpha\beta}\left(\overline{\nabla}_\mu h_{\upsilon\beta} + \overline{\nabla}_\upsilon h_{\mu\beta} - \overline{\nabla}_\beta h_{\mu\upsilon}\right)\right\}$$

$$-\frac{1}{4}\left(\overline{\nabla}_\mu h_\alpha^\beta + \overline{\nabla}_\alpha h_\mu^\beta - \overline{\nabla}^\beta h_{\alpha\mu}\right)\left(\overline{\nabla}_\beta h_\upsilon^\alpha + \overline{\nabla}_\upsilon h_\beta^\alpha - \overline{\nabla}^\alpha h_{\beta\upsilon}\right)$$

$$+\frac{1}{4}\overline{\nabla}_\alpha h\left(\overline{\nabla}_\mu h_\upsilon^\alpha + \overline{\nabla}_\upsilon h_\mu^\alpha - \overline{\nabla}^\alpha h_{\mu\upsilon}\right),$$

$$R^{(1)} = \overline{\nabla}_\mu h^\mu - \Box h - \bar{R}_{\mu\upsilon}h^{\mu\upsilon},$$

$$R^{(2)} = \frac{1}{2}\overline{\nabla}_\mu\left(h^{\alpha\beta}\overline{\nabla}^\mu h_{\alpha\beta}\right) - \frac{1}{2}\overline{\nabla}_\alpha\left\{h^{\alpha\beta}\left(2h_\beta - \overline{\nabla}_\beta h\right)\right\}$$

$$-\frac{1}{4}\left(\overline{\nabla}_\mu h_\alpha^\beta + \overline{\nabla}_\alpha h_\mu^\beta - \overline{\nabla}^\beta h_{\alpha\mu}\right)\overline{\nabla}_\beta h^{\alpha\mu}$$

$$+\frac{1}{4}\left(2h^\alpha - \overline{\nabla}^\alpha h\right)\overline{\nabla}_\alpha h + \frac{1}{2}h^{\alpha\beta}\overline{\nabla}_\alpha\overline{\nabla}_\beta h$$

$$-\frac{1}{2}h_\alpha^\mu\overline{\nabla}_\beta\left(\overline{\nabla}^\alpha h_\mu^\beta + \overline{\nabla}_\mu h^{\alpha\beta} - \overline{\nabla}^\beta h_\mu^\alpha\right) + \bar{R}_{\mu\upsilon}h_\alpha^\mu h^{\upsilon\alpha}$$

$$= \frac{3}{4}\overline{\nabla}_\alpha h_{\mu\nu}\overline{\nabla}^\alpha h^{\mu\upsilon} + h_{\mu\upsilon}\Box h^{\mu\upsilon} - h_\mu^2 + h_\mu\overline{\nabla}^\mu h - 2h_{\mu\upsilon}\overline{\nabla}^\mu h^\upsilon + h_{\mu\upsilon}\overline{\nabla}^\mu\overline{\nabla}^\upsilon h$$

$$-\frac{1}{2}\overline{\nabla}_\mu h_{\upsilon\alpha}\overline{\nabla}^\alpha h^{\mu\upsilon} - \frac{1}{4}\overline{\nabla}_\mu h\overline{\nabla}^\mu h + \bar{R}_{\alpha\beta\gamma\delta}h^{\alpha\gamma}h^{\beta\delta}, \tag{111}$$

where $h_\mu \equiv \overline{\nabla}^\upsilon h_{\mu\upsilon}$. Note that $\bar{g}^{\mu\upsilon}R_{\mu\upsilon}^{(1)} \neq R^{(1)}$, because the latter has additional contribution from $h^{\mu\upsilon}\bar{R}_{\mu\upsilon}$. When total derivative terms are dropped, $R^{(2)}$ makes the contribution to the action

其中 $h_\mu \equiv \overline{\nabla}^\upsilon h_{\mu\upsilon}$ 。注意 $\bar{g}^{\mu\upsilon}R_{\mu\upsilon}^{(1)} \neq R^{(1)}$ ，这是因为后者存在来自 $h^{\mu\upsilon}\bar{R}_{\mu\upsilon}$ 的额外贡献。当去掉全导数项后，$R^{(2)}$ 对作用量的贡献为

$$R^{(2)} \simeq \frac{1}{4}\left(h_{\mu\nu}\Box h^{\mu\nu} + h\Box h + 2h_\mu^2 + 2\bar{R}_{\alpha\beta}h^{\alpha\gamma}h_\gamma^\beta + 2\bar{R}_{\alpha\beta\gamma\delta}h^{\alpha\gamma}h^{\beta\delta}\right). \tag{112}$$

We use the notation $\simeq$ to denote equality up to total derivatives.

我们使用记号 $\simeq$ 表示等式在相差一个全导数的意义下成立。

Heat Kernel Coefficients for $p = 2$ Minimal Operator

对应 $p = 2$ 极小算子的热核系数

For minimal operator $\Delta = -\overline{\nabla}^2 + \mathbf{E}$, the general formulae for any spin are

对于极小算子 $\Delta = -\overline{\nabla}^2 + \mathbf{E}$ ，任意自旋的通用公式如下

$$b_0 = \operatorname{tr}\hat{1},\ b_2 = \frac{1}{6}\bar{R}\operatorname{tr}\hat{1} - \operatorname{tr}\mathbf{E},$$

$$b_4 = \frac{1}{180}\left(\bar{R}^2_{\mu\nu\alpha\beta} - \bar{R}^2_{\mu\nu} + \frac{5}{2}\bar{R}^2 + 6\overline{\nabla}^2\bar{R}\right)\operatorname{tr}\hat{1} + \frac{1}{2}\operatorname{tr}\mathbf{E}^2$$

$$-\frac{1}{6}\bar{R}\operatorname{tr}\mathbf{E} + \frac{1}{12}\operatorname{tr}\Omega^2_{\mu\nu} - \frac{1}{6}\overline{\nabla}^2\operatorname{tr}\mathbf{E}, \tag{113}$$

where $\Omega_{\mu\nu} = \left[\overline{\nabla}_\mu, \overline{\nabla}_\nu\right]$ is the curvature from the covariant derivatives for each spin, and the traces should be taken using the identity $\eta_{\mu\nu}$ for the vector and (13) for the symmetric tensor. The formulae in section "Contribution from the $p = 2$ Nonminimal Vector Operator" are for spin 1, but with an additional nonminimal term.

其中 $\Omega_{\mu\nu} = \left[\overline{\nabla}_\mu, \overline{\nabla}_\nu\right]$ 是各自旋协变导数的曲率，求迹时对向量使用恒等式 $\eta_{\mu\nu}$ ，对对称张量使用 (13) 式。“来自 $p = 2$ 非极小向量算子的贡献”一节的公式对应自旋 1，但多了一个非极小项。

Using these formulae (113) to the Lichnerowicz Laplacians (84), we find, for spin 0 ,

将这些公式 (113) 应用于里奇纳罗维奇拉普拉斯算子 (84)，我们得到对于自旋 0,

$$b_0(\Delta_{L0}) = 1,\ b_2(\Delta_{L0}) = \frac{1}{6}\bar{R},$$

$$b_4(\Delta_{L0}) = \frac{1}{180}\left(\bar{R}^2_{\mu\nu\alpha\beta} - \bar{R}^2_{\mu\nu} + \frac{5}{2}\bar{R}^2 + 6\overline{\nabla}^2\bar{R}\right), \tag{114}$$

and for spin 1

而对于自旋 1

$$b_0(\Delta_{L1}) = d,\ b_2(\Delta_{L1}) = \frac{d-6}{6}\bar{R},$$

$$b_4 = \frac{d-15}{180}\bar{R}^2_{\mu\nu\alpha\beta} - \frac{d-90}{180}\bar{R}^2_{\mu\upsilon} + \frac{d-12}{72}\bar{R}^2 + \frac{d-5}{30}\overline{\nabla}^2\bar{R}, \tag{115}$$

since $\Omega_{\mu\upsilon}A_\alpha = \bar{R}_{\mu\upsilon\alpha\beta}A^\beta \equiv \left(\Omega_{\mu\upsilon}\right)_{\alpha\beta}A^\beta$ and

由于 $\Omega_{\mu\upsilon}A_\alpha = \bar{R}_{\mu\upsilon\alpha\beta}A^\beta \equiv \left(\Omega_{\mu\upsilon}\right)_{\alpha\beta}A^\beta$ 和

$$\operatorname{tr}\Omega^2_{\mu\nu} = \bar{R}_{\mu\nu\alpha\rho}\bar{R}^{\mu\nu\rho\alpha} = -\bar{R}^2_{\mu\nu\alpha\beta}. \tag{116}$$

For spin 2, we find

对于自旋 2，我们得到

$$b_0(\Delta_{L2}) = \frac{d(d+1)}{2},\ b_2(\Delta_{L2}) = \frac{d^2-11d-24}{12}\bar{R},$$

$$b_4\left(\Delta_{L2}\right)=\frac{d^2-29d+480}{360}\bar{R}^2_{\mu\nu\alpha\beta}-\frac{d^2-359d-1080}{360}\bar{R}^2_{\mu\nu}$$

$$+\frac{d^2-23d-48}{144}\bar{R}^2+\frac{d^2-9d-20}{60}\overline{\nabla}^2\bar{R}, \tag{117}$$

since $\left(\Omega^2_{\mu\upsilon}\right)_{\alpha\beta,\rho\sigma}=\bar{R}_{\mu\upsilon\alpha\gamma}\bar{R}^{\mu\upsilon\gamma}{}_{\rho}g_{\beta\sigma}+\bar{R}_{\mu\upsilon\alpha\rho}\bar{R}^{\mu\upsilon}{}_{\beta\sigma}+\bar{R}_{\mu\upsilon\beta\sigma}\bar{R}^{\mu\upsilon}{}_{\alpha\rho}+\ \bar{R}_{\mu\upsilon\beta\gamma}\bar{R}^{\mu\upsilon\gamma}{}_{\sigma}g_{\alpha\rho}$ and

由于 $\left(\Omega^2_{\mu\upsilon}\right)_{\alpha\beta,\rho\sigma}=\bar{R}_{\mu\upsilon\alpha\gamma}\bar{R}^{\mu\upsilon\gamma}{}_{\rho}g_{\beta\sigma}+\bar{R}_{\mu\upsilon\alpha\rho}\bar{R}^{\mu\upsilon}{}_{\beta\sigma}+\bar{R}_{\mu\upsilon\beta\sigma}\bar{R}^{\mu\upsilon}{}_{\alpha\rho}+\ \bar{R}_{\mu\upsilon\beta\gamma}\bar{R}^{\mu\upsilon\gamma}{}_{\sigma}g_{\alpha\rho}$ 和

$$\operatorname{tr}\left(\Omega^2_{\mu\nu}\right)=\frac{1}{2}\left(\{\text{ sum over }\alpha=\rho,\beta=\sigma\}+\{\text{ sum over }\alpha=\sigma,\beta=\rho\}\right)$$

$$=-\left(d+2\right)\bar{R}^2_{\mu\nu\alpha\gamma}. \tag{118}$$

These are the results when the fields do not have any constraints. If the fields have constraint such as transverse, suitable subtraction is needed for spins 1 and 2.

以上是场无任何约束时的结果。如果场存在横向这类约束，则需要对自旋 1 和自旋 2 做适当减项。

Spin 0 does not have any constraint, so their heat kernel coefficients are the same as above. For the transverse spin 1, we have $b_n\left(\Delta^T_{L1}\right)=b_n\left(\Delta_{L1}\right)-b_n\left(\Delta_{L0}\right)$. Hence

自旋 0 不存在任何约束，因此其热核系数与上述结果一致。对于横向自旋 1，我们有 $b_n\left(\Delta^T_{L1}\right)=b_n\left(\Delta_{L1}\right)-b_n\left(\Delta_{L0}\right)$。因此

$$b_0\left(\Delta^T_{L1}\right)=d-1,\ b_2\left(\Delta^T_{L1}\right)=\frac{d-7}{6}\bar{R},$$

$$b_4\left(\Delta^T_{L1}\right)=\frac{d-16}{180}\bar{R}^2_{\mu\nu\rho\sigma}-\frac{d-91}{180}\bar{R}^2_{\mu\nu}+\frac{d-13}{72}\bar{R}^2+\frac{d-6}{30}\overline{\nabla}^2\bar{R}. \tag{119}$$

For the Einstein space, $b_4\left(\Delta_{L_1}\right)$ reduces to

对于爱因斯坦空间，$b_4\left(\Delta_{L_1}\right)$ 化简为

$$b_4\left(\Delta^T_{L1}\right)=\frac{d-16}{180}\bar{R}^2_{\mu\nu\rho\sigma}+\frac{5d^2-67d+182}{360d}\bar{R}^2+\frac{d-6}{30}\overline{\nabla}^2\bar{R}. \tag{120}$$

Similarly those for transverse and traceless spin 2, we have $b_n\left(\Delta^{TT}_{L2}\right)=\ b_n\left(\Delta_{L2}\right)-b_n\left(\Delta^T_{L1}\right)-2b_n\left(\Delta_{L0}\right)$, so

类似地，对于横向无迹自旋 2，我们有 $b_n\left(\Delta^{TT}_{L2}\right)=\ b_n\left(\Delta_{L2}\right)-b_n\left(\Delta^T_{L1}\right)-2b_n\left(\Delta_{L0}\right)$，因此

$$b_0\left(\Delta^{TT}_{L2}\right)=\frac{(d+1)(d-2)}{2},\ b_2\left(\Delta^{TT}_{L2}\right)=\frac{(d+1)(d-14)}{12}\bar{R},$$

$$b_4\left(\Delta^{TT}_{L2}\right)=\frac{d^2-31d+508}{360}\bar{R}^2_{\mu\upsilon\rho\sigma}-\frac{d^2-361d-902}{360}\bar{R}^2_{\mu\upsilon}$$

$$+\frac{(d+1)(d-26)}{144}\bar{R}^2+\frac{(d+1)(d-12)}{60}\overline{\nabla}^2\bar{R}. \tag{121}$$

For the Einstein space, $b_4\left(\Delta_{L2}^{TT}\right)$ reduces to

对于爱因斯坦空间，$b_4\left(\Delta_{L2}^{TT}\right)$ 化简为

$$b_4\left(\Delta_{L2}^{TT}\right)=\frac{d^2-31d+508}{360}\bar{R}^2_{\mu\nu\rho\sigma}+\frac{5d^3-127d^2+592d+1804}{720d}\bar{R}^2$$

$$+\frac{(d+1)(d-12)}{60}\overline{\nabla}^2\bar{R}. \tag{122}$$

Finally we also need the following formulae which also follow from (113):

最后我们还需要以下同样可由 (113) 推导出的公式:

$$b_0(\Delta+aR)=b_0(\Delta),$$

$$b_2(\Delta+aR)=b_2(\Delta)-aRb_0(\Delta), \tag{123}$$

$$b_4(\Delta+aR)=b_4(\Delta)-aRb_2(\Delta)+\frac{1}{2}a^2R^2b_0(\Delta).$$

We can evaluate the one-loop divergent part (42) of the effective action (98) by using Eqs. (114) and (119)-(123).

我们可以利用式 (114) 和 (119)-(123) 计算有效作用量 (98) 的单圈发散部分 (42)。

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